SECRET.

From: Headquarters, Coastal Command.

To: Headquarters Nos: 15, 16, 17, 18, and 19 Group,
" " R.A.F. Gibraltar,
" " R.A.F. Iceland.
All Flying Stations, Squadrons, O.T.U.'s, F.T.U.'s, S. of G.R.
Met. Flights and C.C.D.U.

Ref: ORS/CC/R.235.
Date: 7th July, 1943.

NAVIGATION IN AIR ESCORTS TO CONVOYS.

The attached report dealing with the navigational aspects of escorting convoys for the period 1st. March to the 15th October, 1942 discloses a hitherto unsuspected elliptical distribution of errors of some magnitude.

2. The effective and reliable range of ship-borne D/F exceeds 100 miles and this enables convoys to be met whenever W/T communication is established, thus minimising the effect of the errors in navigation of aircraft and in the predictio[n] of convoys' positions.

3. Nevertheless, every endeavour must be made to reduce the navigational erro[r] of aircraft by the means outlined in the report and especially by the use of astr[o] the importance of which is clearly demonstrated.

Air Vice Marshal,
Senior Air Staff Officer,
COASTAL COMMAND.

S E C R E T.

From:- Headquarters, Coastal Command.

To:-

Date:- 7th July, 1943.
Ref:- ORS/CC/R.235.

NAVIGATION IN AIR ESCORTS TO CONVOYS.

The attached report dealing with the navigational aspects of escorting convoys for the period 1st. March to the 15th October, 1942, has been prepared by Professor H. H. Plaskett and is considered to be most interesting and valuable.

2. Instructions have been issued to Units to put into effect the lessons derived therefrom.

Air Vice Marshal,
Senior Air Staff Officer,
COASTAL COMMAND.

SECRET

ORS/CC Report No.235B.
Copy No..37....

NAVIGATION IN AIR ESCORTS TO CONVOYS.*

Introduction. The probability that an air escort meets its convoy depends upon the accuracy with which the aircraft is navigated to the predicted position of the convoy, the proximity of the convoy to that position, and the method by which the aircraft conducts its search. In an earlier report (ORS/CC Report No.182) it was established that for the period July to December 1941 the probability of meeting decreased from some 84 per cent at 200 miles to 50 per cent at 600 miles from base, and it was concluded that a considerable proportion of these failures was attributable to inaccuracy of aircraft navigation. No information was available, however, to the compilers of this particular report on either the standard or the accuracy of the navigation maintained by the aircraft taking part in these sorties, and it was therefore not possible to indicate how the navigation could be improved, nor what effect such improvements would have on the probability of meeting. There is, therefore, need for a further investigation of the problem, and in the present report, which covers the period from 1 March to 15 October 1942, are discussed in successive sections the standard of aircraft navigation, its accuracy, the observed and anticipated probabilities of meeting, and the effect on meeting of improved searching and of improved navigation.

Sec. A. Standard of Navigation.

Analysis of Logs. The navigation logs of air escorts to convoys should provide some indication of the standard of navigation maintained on these sorties. The difficulty, however, is to discover an objective, quantitative and rapid method of analyzing these logs; the need for objective and quantitative analysis is obvious, while rapid analysis is required in the present instance on account of staff limitations. The standard method of analysis laid down in A.P.1456 is unsuitable (Ref.1.1), and in place of it the following simple method of scoring has been adopted. Ten marks are given for a perfect log, and these are assigned according to the following schedule:

1 mark	for drift determination made at the rate of 4 per hour.
3 marks	for wind velocities found at the rate of 2 per hour. Velocities estimated from wind lanes and drift are counted as half a determination.
1 mark	for accuracy in computation of wind.
1 mark	for evidence of and accuracy in changing from indicated to true air speed.
1 mark	for evidence of the use of the astro compass.
2 marks	for obtaining one astro position line per hour.
1 mark	for obtaining one loop bearing or W/T fix per hour.

In all 444 logs were available for analysis and these logs formed a large and representative sample (Ref.1.2) of all the sorties carried out during the period 1 March to 15 October 1942. These were marked only for that part of the sortie which extended from the last landfall to arrival in the convoy area. The frequency distribution of the total marks thus obtained is shown in Table 1, where the first column contains the mark (0.25 refers to logs with marks between 0 and 0.5, 0.75 to logs with marks between 0.5 and 1.0, and so on), while the second column gives the number of logs which obtained marks within these limits. The distribution of the marks is approximately normal, and the arithmetic mean mark for the 444 sorties is 3.73.

* Footnote.- A non-mathematical account of ORS/CC Report No.235A

Table I. Distribution of Marks for Logs

Mark	Frequency	Mark	Frequency	Mark	Frequency
0.25	2	2.75	42	5.25	45
0.75	1	3.25	51	5.75	19
1.25	28	3.75	71	6.25	9
1.75	22	4.25	65	6.75	2
2.25	30	4.75	55	7.25	2
				Total	= 444

Some care is needed in the interpretation of this mean mark. While it is true that the average standard of navigation within the Command is only some 37 per cent of that laid down in the foregoing schedule of marks, the schedule itself does not necessarily represent a correct or an ideal assessment. Other schedules can and indeed have been drawn up; for example Group Captain Richardson of C.Nav.O. is now making analyses of logs on a roughly similar, though more elaborate system, which incorporates information from station records. The important thing about all such systems of analysis is not then the mark, but the deficiencies in the standard of navigation which the mark is a means of revealing.

Deficiencies in the Standard of Navigation. For the period under review some six marks on the average have been lost. An examination of the marks in each of the seven categories, in which the schedule is divided, shows that there has been an average loss of two marks for wind velocities, one mark for the use of the astro compass, and of nearly three marks for the use of astro and wireless aids. The loss of marks for wind-velocity determination is a result partly of infrequent wind-finding (in 79 logs, or 18 per cent no winds were found at all), and partly from the use of wind lanes and drift as a method of wind finding. On account of their inaccuracy the latter are counted as only half weight determinations, so that if the method is used frequently it results in a low average mark for wind determination. An examination of the 444 logs shows that this is the case; of 3001 winds found during the outward run, the search and the patrol, 744 were found by multiple drifts, 379 by other orthodox methods, and 1878, or more than 62 per cent, by estimation from wind lanes and drift. The loss of one mark for use of the astro compass is a result of the fact that this check on the accuracy of the pilot's compass has only been applied on 13 of the 444 sorties. Finally in two thirds of the sorties no use was made of astro or wireless aids, while in the remaining third the use was so infrequent that only one mark out of a possible three was scored, leading to an average loss of 2.7 marks. A detailed investigation (Ref.1.23) shows that there is no significant difference in the reported cloudiness on astro and non-astro sorties, and further that some navigators manage to take sights on more than 60 per cent of their sorties. It is reasonable to conclude that the infrequent use of astro navigation is due, not so much to lack of opportunity, as to failure to make observations when the clouds permit.

The log analysis has thus shown that during the period 1 March to 15 October 1942 navigation on the outward run to convoys was characterized by infrequent and inaccurate wind finding, and by so restricted a use of astro navigation, the astro compass and wireless that these aids could have had no effect on the average standard of navigation. In short navigation was carried out in effect wholly by D.R., and this with inaccurate ground speed on account of the use of wind lanes and drift. It is of interest to compare these results with those found by Group Captain Richardson from his more elaborate system of analysis which covers the whole time airborne and refers to the period 1 December 1942 to 28 February 1943; the two sets of results follow:-

1.3.42-15.10.42 1.12.42-28.2.43

This comparison is of interest not-so much, perhaps, because it shows an improvement in navigation in the more recent period, though this is encouraging, but because it shows how readily simple methods of log analysis lend themselves to an assessment of the standard of navigation holding within the Command at any given time. It is of the greatest importance, if navigation is to be kept at its present level, or better improved, that such log analysis should be continuously maintained by C.Nav.O.

Sec. B. The Accuracy of Navigation.

Convoy Errors. The accuracy in aircraft navigation on the run-out to the convoy may be found from the difference in position of the convoy, at the time of meeting, as given by itself and as given by the aircraft in the navigator's log. This difference in position is a Combined Error, due on the one hand to the error in the convoy position, and on the other to the aircraft navigation-error, the latter being the quantity in which we are interested. The convoy position at the time of meeting is found by interpolation between two 0800 positions, the latter being taken from the log of the convoy Commodore and supplied to O.R.S. by the Trade Division, Admiralty. The convoy error, whose properties we must know if we are to isolate the aircraft navigation-error from the Combined Error, therefore arises from two sources, the error in convoy navigation and the error in interpolation. The error in convoy navigation depends upon the error in its last astro-fix and upon the error in D.R. navigation made since that fix; its size may therefore be taken (Ref.2.11) to be independent of the distance of the convoy from the British coast, and its direction to be independent of the direction of the convoy track. The interpolation error arises from the assumption, necessary in the absence of more precise information, that the position of the convoy at the time of meeting may be found by linear interpolation between the two appropriate 0800 positions given in the Commodore's log; this assumption is equivalent to supposing the convoy to have travelled in a straight track at uniform speed between these two positions. Like the error due to convoy navigation, the interpolation error in amount will be independent of the distance of the convoy from the British coast, and in direction be independent of the direction of the convoy track, so that the total convoy error, the vector sum of these two component errors, will likewise have these properties.

Combined Errors. Convoy positions from the Commodore's logs were available for 105 of the 444 logs analysed in Sec.A. The Combined Error, defined to be the difference in the position of the convoy as interpolated from the Commodore's log and as given in the aircraft log, may then be found for each of these 105 cases. After the rejection of four observations (Ref.2.21) the remaining Combined Errors were divided according to distance, as shown in Table II, into three roughly equal groups; the upper half of this table gives in successive columns the group number, the number of sorties on which it is based, the range in aircraft distances flown in this group, the mean range (expressed in units of 100 n.miles), and finally the mean track (true bearing) of the aircraft on its run-out to the convoy. The distribution of Combined Errors for each of these groups is shown graphically in Fig.1, where the origin is in each case the convoy position, as interpolated from its own log, while the small circles give the convoy position as reported by the aircraft in its log. From this figure it will be seen, first that the distribution of Combined Errors is elliptical, and secondly that the size of the elliptical distribution increases with increasing distance of the aircraft from its last landfall. These graphical results may be put in a quantitative form by finding the x and y components of the Combined Errors, where a positive x component lies to the east, and therefore from the mean track of the aircraft as given in Table II lies up the aircraft track, and where a positive y component lies to the north, and therefore across the aircraft track to starboard. The centre of gravity (x_o, y_o) of the elliptical distributions may be found (Ref.2.3) from

$$x_o = \frac{\text{Sum of x components}}{\text{Number of x components}} \qquad y_o = \frac{\text{Sum of y components}}{\text{Number of y components}} \quad \ldots(1),$$

while the standard deviations (root mean square errors) are (Ref.2.3) given by lower half of Table II. It will be noted that the values of x_o and y_o are not significantly different from zero,

Table II. Combined Errors as Function of Distance.

Group	No. Sorties	Distances	R=Mean Dist.	Mean Track
		n.m.		
I	34	90 - 230	1.75	270°
II	34	231 - 310	2.70	269°
III	33	311 - 700	4.26	261°
Group	x_o	y_o	σ_x	σ_y
	n.m.	n.m.	n.m.	n.m.
I	+ 1.2	+ 3.2	± 16.0	± 13.6
II	+ 3.7	- 2.4	± 21.2	± 12.1
III	+ 1.2	- 5.0	± 24.5	± 18.3

or in other words the Combined Errors show no effect of systematic error. The value of σ_x is perceptibly larger than σ_y, numerical evidence for the elliptical distribution shown graphically in Fig.1, while both σ_x and σ_y increase with increasing distance of the aircraft from its last landfall. From these values of σ_x and σ_y, the standard deviations of the Combined Errors along and across the aircraft track, it is possible (Ref.2.31) to compute the ellipses which on a normal distribution of errors should contain 50 per cent and 99 per cent of the Combined Errors. These ellipses are shown in Fig.1, and it will be noted that the Combined Errors show as closely a normal error distribution as would be expected from such small samples.

Aircraft Navigation-Error. The Combined Error is the vector sum of the independent errors due to convoy error and aircraft navigation-error. From the previous discussion it is known that the convoy error is independent of distance and direction, while the aircraft navigation-errorm on the other hand, must increase linearly as the distance flown. This follows from the discussion in Sec.A, since it is known that in effect only D.R. navigation was used in all the sorties made during this particular period. We may therefore write the standard deviations of the Combined Errors along and across the aircraft track in the following form (Ref.2.4)

$$\left.\begin{aligned} \sigma_x^2 &= C^2 + \sigma_1^2 = C^2 + A_1^2 R^2 \quad \text{along a/c track} \\ \sigma_y^2 &= C^2 + \sigma_2^2 = C^2 + A_2^2 R^2 \quad \text{across a/c track} \end{aligned}\right\} \cdots (3)$$

where C is the standard deviation of the convoy error, the same in all directions, while $\sigma_1 = A_1 R$ and $\sigma_2 = A_2 R$ are the standard deviations of aircraft navigation-error along and across the aircraft track; R, of course, is the distance of the aircraft from its last landfall, expressed in units of 100 n. miles. Inserting numerical values from Table II in these expressions and solving by least squares we find

$$\left.\begin{aligned} \sigma_1 &= \pm 5.44\ R \\ \sigma_2 &= \pm 2.65\ R \end{aligned}\right\} \quad (4) \qquad C = \pm 13.2 \text{ n. miles.}$$

Thus, by making use of our knowledge of the nature of convoy errors and aircraft navigation-errors, we have succeeded in dissecting out the latter from the Combined Errors. The numerical values of the standard deviations given in eqn. (4) can be used (Ref.2.31) to calculate the dimensions of the ellipse which will contain 50 per cent of the aircraft navigation-errors. The semi-axis major, which lies along the aircraft track, turns out to be 6 per cent of the distance flown, while the semi-axis minor, across the track

Sec. C. The Probability of Meeting.

Observed Probability of Meeting. Of the 444 sorties, for which aircraft logs have been received and marked, only 7 referred to aircraft which, for one reason or another, returned to base before reaching the convoy area. For the remainder, the percentage of sorties which met their convoys are shown in Table III; succeeding columns in the early part of this table show the distance groups in which the 437 sorties have been divided, the mean range R (expressed in units of 100 n.miles) of the sorties in each group, and the percentage of these which met their convoys. This percentage, p, is a closely linear function of the distance run, and a least squares solution of the data given in Table II yields the result

$$p = 100 - (5.18 \pm 0.33) R \quad \dots (5).$$

The duration of the searches, all but 8 per cent of which were carried out by the standard C.L.A. method, is shown in the latter part of Table III. For the mets this duration is shown by giving the percentage of sorties with searches of 0 hrs., of 0.01 to 1.00 hrs, and greater than 1.0 hrs.; it will be noted that 40 per cent of the sorties, even at the extreme range, meet their convoys without making any search. For the not-mets, the mean duration of the search is shown in the last column of the table. The duration, as well as the effectiveness of the C.L.A. search depends upon the visibility; it is found (Ref.3.1), whether the search is carried out by A.S.V. or by eye, that the visibility may conservatively be estimated to lie between 10 and 15 n.miles.

Table III. Meeting of Convoys.

Range	Total no. sorties	R=Mean Distance	p=Percentage Met	Met Sorties Percentage Sorties with Search Duration			Not-met sorties Mean Search Duration
				0^h	$\cdot 01-1\cdot 00$	$>1\cdot 00$	
n.m.							
<200	160	1.24	88.8	57.6	31.7	10.7	$3^h.12$
200-299	134	2.48	86.6	43.1	41.4	15.5	2.92
300-399	80	3.36	81.2	41.5	40.0	18.5	2.40
>400	63	5.05	76.2	39.6	35.4	25.0	1.90

Error in Predicted Position of Convoy. The observed probability of meeting, given in Table III and eqn. (5), may be compared with the probability to be anticipated from the accuracy of aircraft navigation determined in Sec.B, provided the error in the predicted position of the convoy can be found. To find the latter, the difference between the convoy position as given at 0800 in the Commodore's log, and the predicted position for the same time as given in the Liverpool Form White, may be used. This leads to a Combined Prediction Error, made up of the error due to convoy navigation and the error due to prediction proper, the latter being the quantity in which we are interested. The x and y components of the Combined Prediction Error, x measured along the convoy track in the direction of its motion, and y measured across the convoy track to the port beam, may be used to find the centre of gravity of the distribution of these errors and their standard deviations. The results, derived from eqns. (1) and (2), are shown in Table IV for 50 inward and 33 outward bound convoys. From the values of x_o and y_o, it will be noted that there is a systematic error, the convoy being predicted to lie some 12 n.miles further from the coast than its position as given by the convoy Commodore; further it will be noted, since the standard deviation, σ_x, along the convoy track, is nearly three times as large as σ_y, across the track, that the distribution of these Combined Prediction Errors is highly elliptical. More detailed investigation (Ref.3.21) shows that the accidental part of this Combined Prediction Error is independent of the air-distance of the convoy from the British coast, so that a separation of the prediction error proper can only be effected by an indirect method (Ref.3.21). The means of upper and lower limits to the standard deviations of prediction error proper are then

Table IV. Combined Prediction Error.

Convoy	No.	x_o	y_o	ρ_x	ρ_y
Inward	50	-11.5 ± 3.0	+ 1.0 ± 0.9	± 31.4	± 9.8
Outward	33	+11.2 3.2	- 5.0 1.4	± 26.8	± 11.8

Anticipated Probability of Meeting. The anticipated probability of meeting a convoy, knowing the area covered by the C.L.A. search, may be found from the following considerations. With origin as the predicted position of the convoy, and x and y axes directed respectively along the convoy track towards Great Britain and to the left of the track, the actual positions of the convoy show a normal elliptical distribution, with standard deviations given by eqn. (6), and this distribution is centred on a centroid displaced from the origin by the systematic errors of prediction, with the appropriate signs. An aircraft flies out on escort duty and at E.T.A. predicted position has, because of its own navigation error, co-ordinates x', y'; about this position it carries out a standard C.L.A. search. The probability that the convoy lies within the area of this search, centred on x', y', may immediately be calculated from the normal elliptical distribution of convoy prediction errors. The probability, however, that the aircraft arrives in an elementary region centred on x', y' is given by the normal elliptical distribution of aircraft navigation-errors with standard deviations given by eqn. (4). The product of these two independent probabilities is then the probability of meeting the convoy when the aircraft arrives at x', y'; but the aircraft may arrive at any point, and consequently the total probability of meeting is the sum of the separate probabilities of meeting, one corresponding to each point of arrival of the aircraft.

Details of the mode of calculation, which is exact, are given elsewhere (Ref.3.3), and here it will suffice to give the results which, for inward convoys, are summarised in Table V. The table is divided into two parts, the first half for the standard C.L.A. search carried out with a visibility (A.S.V. or eye) of 10 n.miles, and the second half for the same search with a visibility of 15 n.miles. The second column of the table gives the distance, R, of the aircraft, in units of 100 n.miles, from its last landfall, the third column gives the anticipated probability of meeting, P_1, calculated by the above methods, while the final column gives the observed probability of meeting from eqn. (5). The anticipated probability of meeting, P_1, is not only definitely smaller than that observed, but also shows little dependence on the distance of the aircraft from its last landfall. A clue to the origin of this discrepancy is given in Table III, where it is shown that on the average well over 40 per cent of the sorties meet their convoys without any searching. This must be a consequence of the practice of flying along the track of the

Table V. Anticipated Probability of Meeting (Inward Convoys)

Vis.	R	P_1	P_2	$P=P_1+P_2$	Obs. Prob. of Meeting
10 n.m.	1.50	0.550	0.283	0.833	0.922
	3.00	.521	.254	.775	.845
	4.50	.591	.123	.714	.767
	6.00	.491	.112	.603	.690
15 n.m.	1.50	0.560	0.372	0.932	0.922
	3.00	.542	.338	.880	.845
	4.50	.643	.170	.813	.767
	6.00	.552	.159	.711	.690

convoy for some distance prior to reaching the initial point of the C.L.A. search. During this part of the run, which may extend 100 n.miles or more to landward of the predicted position, the aircraft is automatically carrying given in the fourth column of Table V, and it will be noted that there is a quite substantial probability of meeting before the commencement of the C.L.A. search. The total probability of meeting, P, is the sum of the mutually exclusive probabilities P_1 and P_2, and its value is given in the fifth column of Table V. Comparing this anticipated probability of meeting with that observed, it is evident that there is excellent agreement for a visibility distance between 10 and 15 n.miles. This agreement affords satisfactory evidence of the accuracy of the aircraft navigation-errors found in Sec.B, and the prediction errors found in this section; it is therefore legitimate to use these results to find the effect of improved searching and improved aircraft navigation on the probability of meeting.

Sec. D. The Probability of Meeting for Improved Searching and Navigation.

Constant-Probability Search Area. It is of some interest to find the size of the area which must be searched in order to have a constant probability of meeting, irrespective of the distance of the aircraft from its last landfall. Assume that the aircraft navigation-errors are as found in Sec.B, and that apart from the systematic error in prediction, which can readily be removed as soon as the predictors know of its existence, the accidental errors of prediction remain as found in Sec.C. Then by an application of the methods described above (Ref.3.41), we may calculate the area over which a search must be extended to have a probability of meeting of 0.95, that is one convoy in 20 not met. The results are given in Table VI for two cases, one where the aircraft track is parallel to that of the convoy, as is the case for convoys HX, SC, ON, OS and HG, SL, OG in the Bay, and the other where the aircraft track is perpendicular to that of the convoy, as is the case for convoys HG, SL, OG when met west of Ireland. The first column gives the distance R of the aircraft from its last landfall, expressed in units of

Table VI Search Area for $P = 0.95$

R	A/C Track ‖ Convoy		A/C Track ⊥ Convoy	
	‖ Convoy	⊥ Convoy	‖ Convoy	⊥ Convoy
	n.m.	n.m.	n.m.	n.m.
1.50	67	23	65	27
3.00	74	27	67	42
4.50	84	34	69	59
6.00	97	41	73	76
7.50	111	49	78	94
9.00	127	57	84	112

100 n.miles, while the second and third columns give the distance, respectively parallel and perpendicular to the convoy track, on either side of the predicted position, over which the search must be extended in order to have the required probability of meeting; these two columns refer to parallel aircraft and convoy tracks, while the fourth and fifth columns give similar dimensions for perpendicular aircraft and convoy tracks. From the table it will be seen how sensitive are the dimensions of the search area to the circumstances of meeting; not only do the dimensions increase rather more rapidly than proportionately to the distance flown by the aircraft, but in the case of perpendicular aircraft and convoy tracks the search area changes from a rectangle along the convoy track, to a square, and then to a rectangle perpendicular to this track as the distance flown increases. These changes in the constant-probability search area are of course a direct consequence of the linear increase in the aircraft navigation-error with the distance flown

of D.R. the search must be conducted on straight tracks, there appear to be two main forms, one the C.L.A. search and the other a form of track search suggested by Group Captain Richardson. These are shown in Fig.2 for a rectangular search area with semi-dimensions of 100 by 40 n.miles (corresponding closely with the P = 0.95 search-area at a distance of 600 n.miles, for parallel tracks), the searches being assumed to be conducted with a visibility of 15 n.miles. By a simple extension of the methods outlined in Sec.C it is possible to compute the anticipated probability of meeting as a function of the time from the commencement of the search; these anticipated probabilities for each of the two methods of search are shown in Fig.3, the ordinate being the probability of meeting and the abcissa the time from the commencement of the search. The computation is carried out for the aircraft navigation-error and the accidental errors of prediction found in Secs.B and C. From the figure it will be seen, as should indeed be the case, that by the end (Fig.3) of the search each method achieved a probability of meeting of 0.95, but, while the C.L.A. search creeps home first, the track search for the greater part of the period has a far higher probability of meeting. Thus at the end of one hour the track search has a probability of meeting of 0.42 as contrasted with 0.08 for the C.L.A. search, while at the end of two hours the respective probabilities are 0.64 and 0.37. In this case the race is to the swift, and there can be no doubt that the track search provides the more efficient method of finding convoys. Elsewhere (Ref.3.41) details are given of two simple and practical track searches, designated by the letters A and B which cover aircraft ranges respectively from 150 to 450 n.miles, and from 450 to 750 n.miles, and which give probabilities of meeting in excess of 0.90.

Effect of Improved Navigation. While the track search, described above, attains a high probability of meeting near its commencement, it is important to note that the anticipated probability of 0.95 will only be reached provided the search is, when necessary, actually completed. Assuming a speed of 125 k. and a visibility distance of 15 n.miles, the durations of the completed track searches A and B are respectively 2.44 and 4.92 hours, while if the visibility is 10 n.miles the corresponding times are 3.68 and 6.71 hours. Improved searching is thus a somewhat illusory method of increasing the probability of meeting, for the gain is only made at the cost of a corresponding reduction in the time available for escort duty. A high probability of meeting, together with a large fraction of the endurance available for escort duty, can only be secured if, combined with the track search, there is a basic improvement in aircraft navigation.

While limited improvement in aircraft navigation can be immediately effected by the use of multiple drift winds and by the full use of the two minute integrating Mk.IXA Sextant (Ref.3.42), the most substantial improvement would result from a modification of the Mk.IXA Sextant to eliminate the observer's personal equation. This may be expected (A.&A.E.E. Res./172) to give a single position line with a 50 per cent accidental error of ± 1.6 n.miles, and a systematic error of 1.0 n.miles, or since these errors are independent the average 50 per cent error would be $\sqrt{(1.6)^2+(1.0)^2}$ = ± 1.9 n.miles, equivalent to a standard deviation of ± 2.8 n.miles. This error is independent of the distance of the aircraft from land, so that astro fixes from two perpendicular position lines will show a circular distribution of errors with $\sigma_1 = \sigma_2$ = ± 2.8 n.miles. The rectangular search area at any distance for a 0.95 probability of meeting will have semi-dimensions of 64.5 by 15.2 n.miles, respectively along and across the convoy track (for parallel tracks), so that track search B could be replaced by a search taking approximately 1.2 hours for visibility 15 n.miles, or 1.7 hours for a visibility of 10 n.miles. These durations are comparable with those now taken by the standard C.L.A. search, so that as much time would be available for useful escort duty as at present. Aircraft with improved astro-navigation would, however, have a probability of meeting of 0.95, as contrasted with 0.69 at 600 n.miles given by the present standard of navigation.

The considerable gain in efficiency which improved astro-navigation makes possible will only be realised, provided the limitations on the use of astro are not too crippling. These limitations arise from the circumstance that, on over one half of the day sorties, only single position lines can be obtained (the moon not being available), and from the fact that astro-sights cannot be taken in ten-tenths cloud. The effect of the first of these limitations

limitations there may be, due to cloud, could be almost wholly eliminated if the operational height of aircraft were raised to 5000 feet, that is above the top of the lower lying cloud layer.

Assume that of 100 sorties with improved astro navigation available, the effect of these limitations would be to necessitate the use of D.R. navigation alone on 33 sorties, - a safe upper limit. Then the total number of mets would be 67 x 0.95 + 33 x 0.69 = 86.3, where 0.95 is the probability of meeting for improved astro-navigation, and 0.69 is the probability for D.R. navigation (eqn.5 for R = 600 n.miles). To get this same number of mets with D.R. navigation alone, as is the case at present, 100 x 86.3 ÷ 69.0 = 125 sorties would have to be made, or the minimum gain in efficiency from the use of improved astro-navigation would be 25 per cent.

Conclusions and Recommendations.

It has been shown that:-

1) The standard of navigation within the Command for the period 1 March to 15 October 1942 is characterized by infrequent wind determination, and by the use of the inaccurate method of wind lanes and drift. Astro and wireless aids to navigation are used so infrequently as to be on the average ineffective in improving the results of D.R.

2) The error of aircraft navigation is found by a comparison of the convoy position as given by the convoy Commodore and as found by the aircraft navigator from his D.R. One half of the errors are contained within an ellipse of semi-axis major, lying along the aircraft track, equal to six per cent of the distance flown, and of semi-axis minor of three per cent of this distance. The greater inaccuracy along the track is evidence that the ground speed is known less accurately than the track, and is what might have been anticipated from the infrequent and inaccurate wind finding.

3) The observed probability of meeting a convoy decreases linearly as the distance flown, and at 100 n.miles from the last landfall is 95 per cent, at 600 n.miles, 69 per cent. The anticipated probability of meeting, exactly calculable from the known aircraft navigation-error, from the error in prediction and from the method of search, agrees very satisfactorily with that observed, and thus confirms the accuracy of the aircraft navigation-error.

4) A track search gives a higher probability of meeting in its earlier stages than does the C.L.A. search. A probability of meeting of 0.95 by either method of search is, however, only attainable at the expense of time otherwise available for escort duty, and in order to retain both a high probability of meeting and the maximum time for escort duty, it is necessary to improve the standard and accuracy of aircraft navigation. Improved astro-navigation, full allowance being made for the limitations in its use, would give a 25 per cent increase in the efficiency of medium long-range escorts to convoys

It is recommended that

1) A continuous check on the standard of navigation within the Command be maintained, preferably by the methods at present in use by C.Nav.O.

2) The existing C.L.A. search be abandoned, and standard track searches be substituted.

3) In order to get the highest possible accuracy in astro-navigation, the personal equation of the observer be eliminated, either by the introduction of a reversing prism and gear in the Mk.IXA Sextant, or less satisfactorily, particularly for sights of the sun and moon

once the sights are taken, of using them to correct the D.R. navigation.

K.E.B./H.H.P./ORS/CC.
16.4.43.

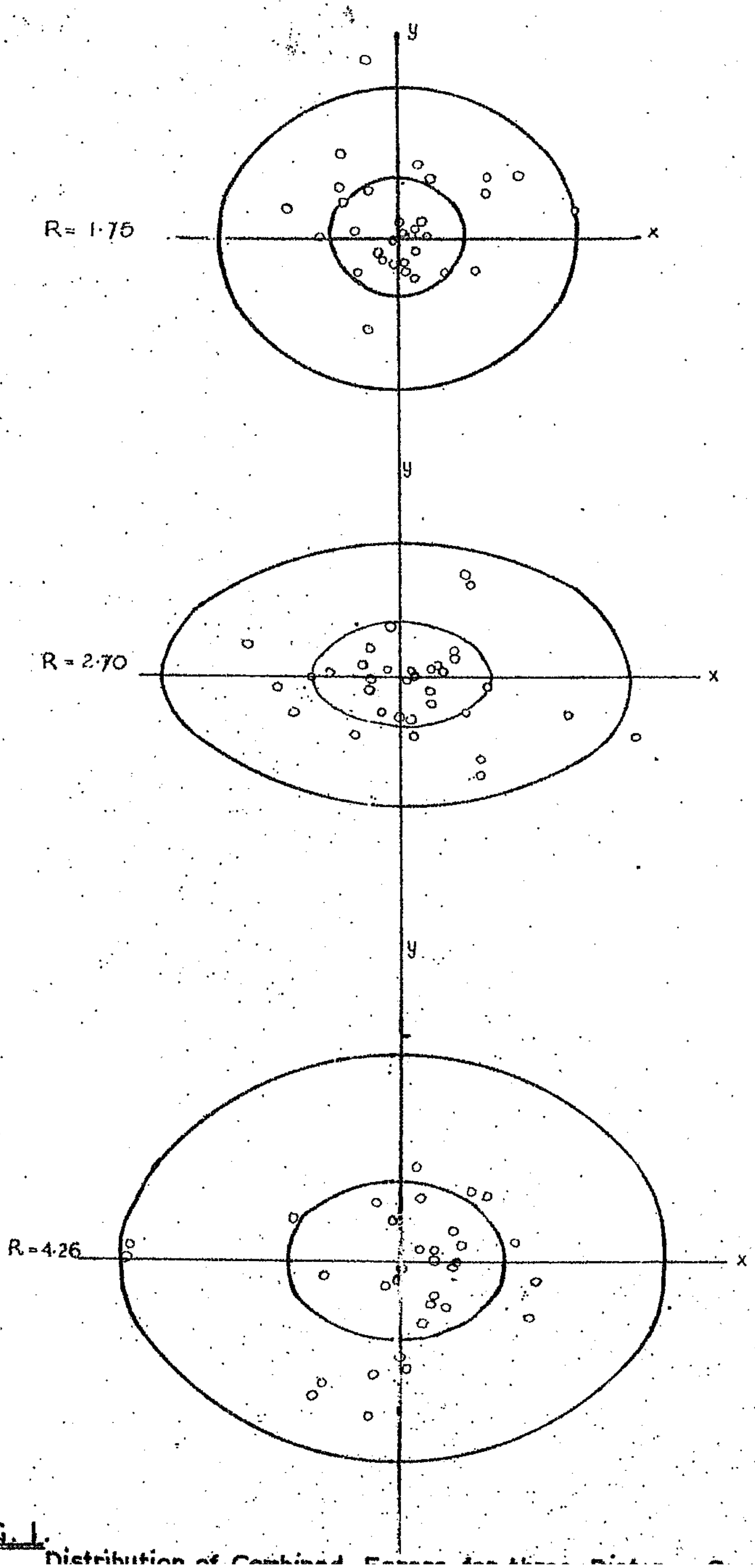

FIG. I.
Distribution of Combined Errors for three Dist

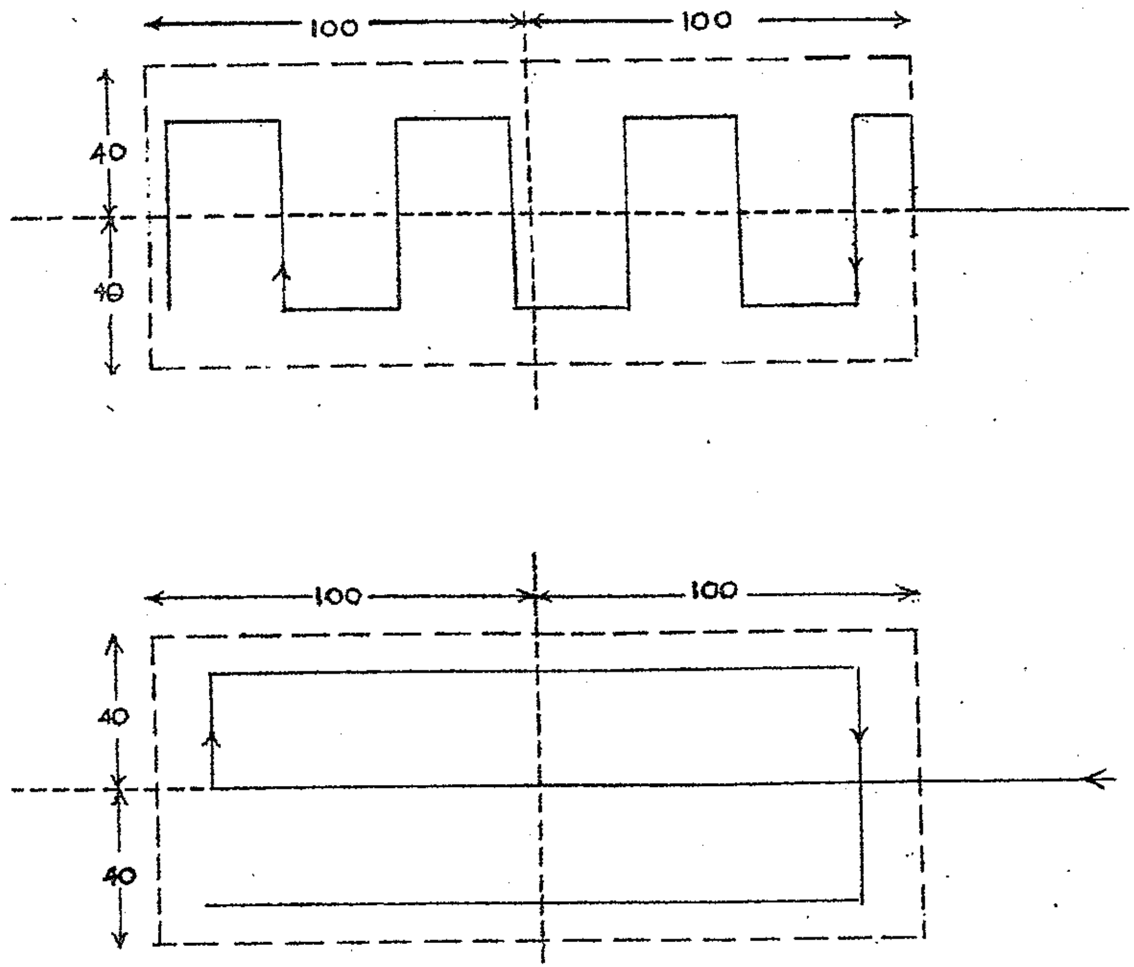

FIG 2. C.L.A. SEARCH (upper figure) and TRACK SEARCH (lower figure) for $P = 0{\cdot}95$ at 600 n.miles and VISIBILITY ~ 15 n.miles.

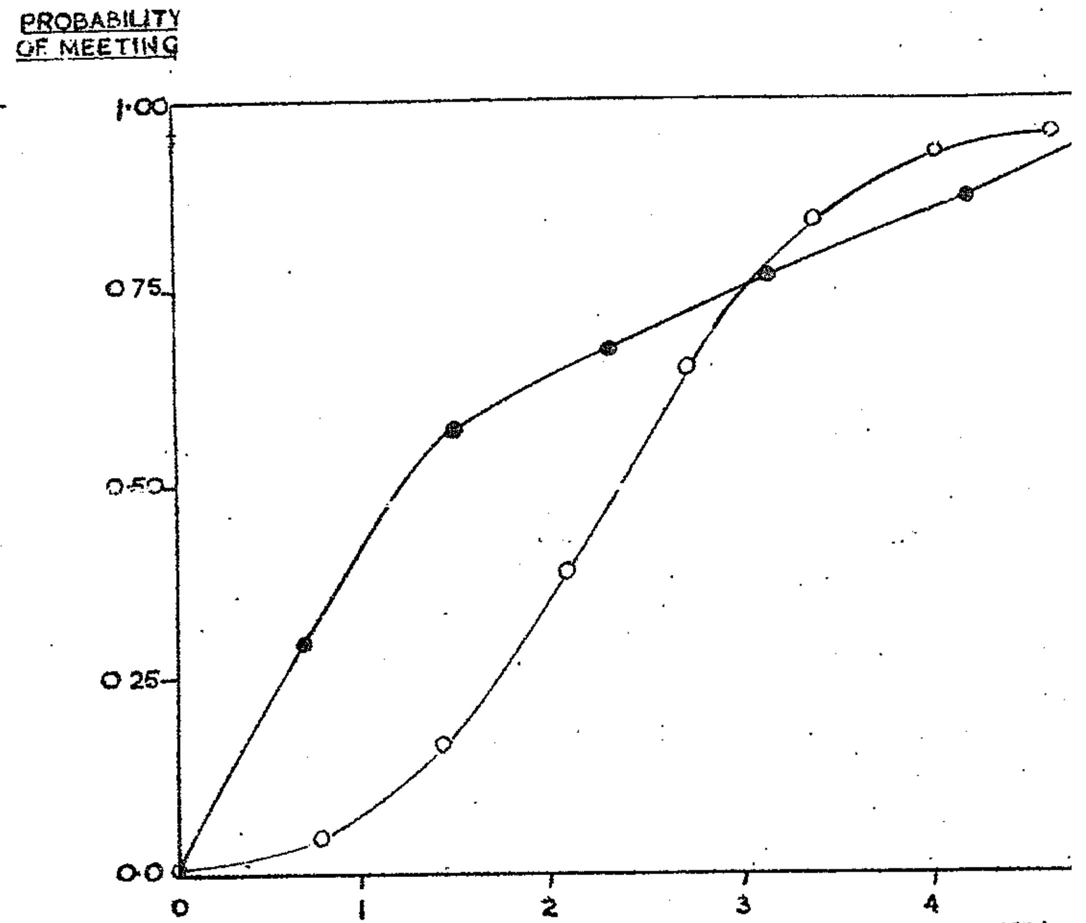

FIG. 3. PROBABILITY of MEETING as a FUNCTION OF LENGTH TIME SPENT ON C.L.A. SEARCH (open circles) AND ON TRACK SEARCH (filled circles). FIGURE REFERS TO THE TWO SEARCHES SKETCHED IN FIG 7.

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NAVIGATION IN AIR ESCORTS TO CONVOYS

SUMMARY

Standard of Navigation. A simple method of scoring has been applied to the navigation logs of 444 air escorts to convoys during the period 1 March to 15 October 1942; these particular sorties form a large and representative sample of all the air escorts during this period. The average mark is 37 per cent of the possible attainable for a not unduly high standard of navigation. Analysis of these marks shows that the low scoring is attributable to a poor determination of winds, over 60 per cent of which were found by the inaccurate method of wind lane and drift, and to an almost complete neglect of astro and wireless aids to navigation.

Accuracy of Navigation. The difference between the position of the convoy, when met, as given by the aircraft navigator in his log and as given by the convoy Commodore in his log, is due to the error both of aircraft navigation and convoy navigation. Since the former varies as the distance of the aircraft from its last landfall, while the latter is independent of this distance, it is possible to find the error due to aircraft navigation alone. These errors show an elliptical distribution; the semi-axis major of the ellipse which contains one half the errors lies along the track of the aircraft and has a length equal to six per cent of the distance flown; the semi-axis minor, across the track, has a length of three per cent of this distance. The errors show a normal distribution and are such as would have been anticipated from the standard of navigation maintained during these sorties.

Probability of Meeting. The percentage of air escorts, which failed to meet their convoys during the period concerned was 5 per cent at 100 miles and increased linearly as the number of 100 miles flown by the aircraft from its last landfall. Such failure is due to errors in the predicted position of the convoy, to errors in aircraft navigation, and to the methods of search. An examination of the predictions shows that, apart from a systematic error which varied with the distance of the convoy, the accidental errors, which are independent of distance, have a highly elliptical distribution with standard deviations of 29 miles along the convoy track and 9 miles across it. Knowing the distribution of errors for aircraft navigation and for convoy prediction, it is possible exactly to calculate the probability of meeting a convoy for any given mode of searching. Agreement of the observed with the calculated frequency of meeting is obtained for the standard Creeping Line Ahead search, provided allowance is made for the fact that the aircraft runs up to the predicted position along the track of the convoy, and this agreement may be taken as evidence of the accuracy with which the aircraft navigation and the convoy prediction errors have been determined. The method of calculating the probability of meeting is then applied first to show the advantages of a track search, suggested by Group Captain Richardson, over the C.L.A. search, and secondly to show the effects of improved air navigation. It is concluded that a modified form of astro-navigation, applied on all possible occasions, would increase the efficiency of air escorts to convoys by at least twenty five per cent, and since this would be equivalent to a corresponding increase in the number of long-range aircraft at present attached to Coastal Command, it is recommended that this modified form of astro-navigation be adopted at the earliest possible date.

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NAVIGATION IN AIR ESCORTS TO CONVOYS

Introduction

Accurate navigation is a pre-requisite to success in all the activities of Coastal Command. In none is this more true than in air escort to convoys where, before the patrol can be commenced, the convoy must be intercepted, on occasion as much as 1000 miles from the nearest land. The remarkable success of these escorts in driving the U-Boats from the Western Approaches is evidence of the accuracy with which Coastal Command aircraft are navigated over short distances; what is now required is that still higher degree of accuracy in navigation without which convoys at medium and long range cannot be met, let alone protected.

In a previous report (Ref.1), covering the period July to December 1941, it was shown that the percentage of convoys which were met decreased from 84 per cent at 200 miles to approximately 50 per cent at 600 miles from the aircraft base. From a study of the errors in the predicted positions of the convoys it was concluded that a large part of these failures could be attributed to errors in aircraft navigation, estimated as 5 per cent of the distance run, but as no study could be made of either the standard or accuracy of aircraft navigation maintained during these sorties, no specific recommendations could be advanced for its improvement.

The present report is concerned almost wholly with this question of the standard and accuracy of aircraft navigation, the period covered being that from 1 March to 15 October 1942. The standard of navigation is examined in the first section by a study of a large sample of navigation logs, provided through the co-operation of the Navigation Section of Coastal Command. The accuracy of navigation is found in the second section by means of the differences in position of the convoy as given by the aircraft navigator and the convoy Commodore, the logs of the latter being provided by an arrangement with the Board of Trade through the co-operation of Dr. J.H.C. Whitehead. The third and final section is concerned with the actual and calculated probabilities of meeting, and this comparison leads to recommendations for a change in the mode of searching and in the methods of navigation.

Sec.1. The Standard of Navigation.

1.1. The Method. The standard method of log analysis is that laid down in A.P.1456 (Chap.xxx). In this it is shown that the Final Error in the position of the aircraft at E.T.A. terminal point may, provided the winds prevailing over the legs flown are accurately known, be analysed into what are termed Calculation Error, Wind Change Error and Other Error. A re-examination of the proof given in A.P.1456 shows that Other Error is due to failure to fly the aircraft on the courses and at the speeds and heights required by the navigator, and might more appropriately be termed the Pilot's Error. This mode of analysis is, however, unusable, unrepresentative and meaningless. Unusable because under operational conditions Final Errors are infrequently, and accurate winds never, known; unrepresentative because the analysis can only be applied to perfectly kept logs so that the average standard of navigation prevailing in the Command would be assessed at too high a level; and meaningless because errors in calculation, errors in instruments and their use, errors in judgment by the navigator and mistakes by the pilot have all contributed to each of the analysed Wind Change Error and Other (Pilot's) Error, so that no insight into the actual cause of failure of the navigation can be gained by this method.

Without any knowledge, however, of either the Final Error or the winds, it is possible from an inspection of the log to ascertain the nature of the as follows:-

1 mark	for drift determinations made at the rate of 4 per hr.
3 marks	for wind velocities found at the rate of 2 per hr. Velocities estimated from wind lane and drift are counted as half a determination.
1 mark	for accuracy of wind computation as checked on a Navigational Computer from the data supplied in the log. The full mark is assigned provided the wind given in the log agrees with the re-computation within 5 knots and 10° in direction.
1 mark	for evidence and accuracy of the change from indicated to true air speed.
1 mark	for evidence of the use of astro compass to check the magnetic compasses.
2 marks	for obtaining one astro position line per hr.
1 mark	for the use of any wireless aid (loop bearing, M.F./D.F. bearing or fix, etc.) per hr.

These marks were assigned only for that part of the sortie from the last landfall to the expected meeting point with the convoy. For convenience in subsequent discussion they were entered on a card, one space for each mark, and one card for a log. In addition the card carried the squadron number, the aircraft number and the names of the captain and the navigator; there were spaces for the day and the time of flight, the distance from the last landfall to the predicted position of the convoy, the duration of the flight over this distance, including that part carried out in darkness, a set of spaces for marking the search, and a summary of the meteorological conditions as reported by the aircraft and given in the Forms Orange. This method of marking proved to be both objective and rapid. While all the logs were marked by one of the writers, at least a quarter were marked independently by the other: the two sets of marks were in good agreement. The average time required to mark a log was of the order of 15 minutes, though the completion of the card, involving as it did a consultation of Forms Orange, Forms Liverpool White and Group Narratives required a considerably longer time.

1.2. <u>The Results of Log Marking.</u> The period covered in this analysis is 1 March to 15 October 1942. During this period the total number of sorties to convoys, excluding those made by Hudson aircraft, was 947. Logs were received for 43.4 per cent of these sorties; of the logs received 81.9 per cent referred to aircraft which met their convoys, while in the total of 947 sorties 79.2 per cent, or practically the same fraction, referred to successfully meeting aircraft. Consequently if meeting be regarded as a criterion of good navigation, the sample of logs received was representative of the navigation carried out during this period.

In all 444 logs were available for marking. Of these sorties 39 were made by Hudsons, 39 by Whitleys, 212 by Sunderlands, 8 by Catalinas, 70 by Fortresses, 72 by Liberators and 4 by Lancasters. The distribution of marks received by these logs is shown in Table 1; the first column contains the midpoint of the mark interval, thus a mark of 0.25 (more accurately 0.245) refers to logs which had marks between 0 and 0.49, a mark of 0.75 to logs with marks between 0.50 and 0.99 and so on, while the second column gives the number of logs with marks within these limits. The distribution of the marks is approximately normal with an arithmetic mean of 3.73 and a 50 per cent error of a single deviation from this mean of ± 0.88. The navigational standard required to reach the possible mark of 10 is not unduly high, and it is of some importance to see why the average mark is only some 37 per cent of this.

1.21. <u>Failures in D.R. Navigation.</u> Referring back to the method of marking, it will be seen that 7 out of the possible 10 marks are given for D.R. navigation alone. Reasonable proportions of the possible marks for drift taking, for accuracy and for conversion of indicated to true air speed were obtained in the majority of logs. The outstanding failure was in wind finding for which 3 marks were allotted. Thus in 79 logs, or 17.8 per cent

6

Table I. Distribution of Marks for Logs.

Mark	Frequency	Mark	Frequency	Mark	Frequency
0.25	2	2.75	42	5.25	45
0.75	1	3.25	51	5.75	19
1.25	28	3.75	71	6.25	9
1.75	22	4.25	65	6.75	2
2.25	30	4.75	55	7.25	2
				Total =	444

76 per cent made in daylight throughout the average mark was 1.22. In addition to infrequent wind determination the low mark for wind finding was also caused by the use of wind lane and drift, which for purposes of marking was counted as only half a determination. Thus of the 3001 winds found in all the 444 sorties over the period covered by the outward run, the search and the ensuing patrol, 744 were found by multiple drift, 379 by other orthodox methods, and 1876, or more than 62 per cent, by estimation from wind lane and drift. This enumeration if anything under-estimates the proportion of winds found by wind lane and drift in the outward run, since it includes the period of search and patrol when the majority of multiple drift winds are found. The half mark given for such determination is, if anything, an over-estimate of their value, partly because of the uncertain correction for change of wind direction with height (incidentally many navigators make no correction), and partly because of the possibility of sideslip vitiating the resulting wind velocity.

In addition to the average of two marks lost for infrequent or inaccurate wind finding, there was a loss of very nearly one mark for the use of the astro compass. This simple aid which permits a quick check on the accuracy of the magnetic compass, especially necessary in view of the difficulty of making ground swings of flying boats, was only used 13 times in the 444 sorties, or on 2.9 per cent of the possible occasions.

1.22. Failures in the Use of Navigational Aids. The remaining loss of marks occurred from failure to use the navigational aid, provided by the sextant and by wireless. In 294, or roughly two thirds of the sorties, no use was made of these particular aids, while in the remaining 150 the average mark scored was only one out of a possible three - equivalent to a loss on the average of nearly 2.7 marks. Failure to get loop bearings or D.F. fixes is wholly due to failure by the navigator to get such information from the wireless operator, but failure to get astro sights may be occasioned by cloudy weather. Thus while one or other, or both, aids were used on 150 sorties, astro sights were only taken on 93 sorties. This suggests that the weather may be the limiting factor in the use of astro, and consequently this question must now be examined.

1.23. Percentage of Cloud on Astro and non-Astro Sorties. The meteorological conditions in the convoy area are reported in the Forms Orange, and refer to the conditions prevailing at the time as observed from the aircraft. For the purposes of this discussion these conditions may be taken as those holding throughout the outward run to the convoy area. The fraction of the sky covered by cloud is given in Table II; the first column contains the mid point of the interval of cloud-covered sky, thus 0.1 includes those sorties with cloud ranging from 0.0 up to 0.19, while the second and third columns give the frequency of astro and non-astro sorties on which this fractional cloudiness was observed. The total number of sorties included in the Table is 430; for 14 sorties no meteorological reports were available. It will be noted that the distribution of the reported fraction of the sky covered with cloud is in both cases markedly J-shaped. From meteorological studies it is well known that such distributions are usually U-shaped, and it may be concluded that the failure to find such a distribution in the present material is a consequence of a systematic over-estimate of cloudiness on the

Table II. Fraction of Cloud-Covered Sky.

Fraction Cloud	Frequency	
	Astro	Non-Astro
0.1		9
0.3	16	23
0.5	15	51
0.7	11	46
0.9	51	208
Totals	93	337

The frequency distributions of reported cloudiness for astro and non-astro sorties are shown graphically in Fig.1. The data for this figure are taken directly from Table II, but, in plotting, the frequency ordinates for the non-astro sorties have been reduced in the ratio of 93/337 in order to make the areas under the two distributions equal. The figure shows that the distributions of reported cloudiness differ in that the astro sorties have fewer cases of cloudiness 0.9, and more cases of cloudiness 0.3, than the non-astro sorties. This is very much what would have been expected but as the differences are small it is open to question whether they may not have resulted from random sampling. A somewhat crude test can be made by regarding the distribution of reported cloudiness for the non-astro sorties as the hypothetical distribution; then a calculation of Pearson's X^2 for goodness of fit shows that the probability is 1 in 12 that the observed distribution of cloudiness on astro sorties resulted from random sampling of the hypothetical distribution. This is too large a probability to be regarded as indicative of a real difference between the two distributions of cloudiness, and we must conclude that there is no significant difference in cloudiness in the two types of sortie. This view is confirmed by the fact that the reported mean heights of the cloud base and the cloud top are respectively 2000 and 4400 feet for the astro sorties, and 1800 and 4800 for the non-astro sorties; in other words as regards height and thickness of cloud the two types of sortie are essentially identical, and this suggests a similar identity for the percentage of cloudiness.

The evidence does not suggest any real difference in cloudiness on the two types of sortie. The failure to take astro sights on 351, or 79 per cent of the sorties, is then to be attributed more to a failure of the navigator to make use of his opportunities than to clouds denying him that opportunity. This view is to some extent confirmed by an examination of the squadrons and the navigators responsible for the astro sights which were obtained. In Table III are listed according to squadron the total number of sorties made, the number of these in which astro sights were taken, and finally for those squadrons with a total of 20 or more sorties the percentage of these in which astro was used. The striking feature of the table is the high percentage of astro sorties made by 120 and 201 Squadrons. Even more striking is the fact that of the 189 sorties made by 34 navigators in these two squadrons, 10 navigators alone were responsible for 52 out of the total of 67 astro sorties made; amongst these 10 navigators one took astro sights on 75 per cent of his sorties, three on more than 60 per cent of their sorties, one on more than 50 per cent, two on more than 40 per cent, while the remainder took sights on more than 30 per cent of their sorties. It is not unlikely that what was done by these 10 navigators could equally well have been done by all the navigators in the 19 squadrons concerned; it was because of failure to seize the opportunities, rather than to cloudiness, that so few astro sights were taken.

Table III. Percentage of Astro Sorties per Squadron

Sqdn	Total Sorties	Astro Sorties	%	Sqdn	Total Sorties	Astro Sorties	%	Sqdn	Total Sorties	Astro Sorties	%
10	22	3	14	160	7	4		423	5		
44	4	2		201	124	42	34	461	1		
48	5			206	7			500	17		
51	4			210	8	3		608	4		
56	24	1	4	220	70	4	6	612	7	1	
77	4			224	6	1					
120	65	25	38	228	60	7	12				
								Totals	444	93	

1.3. Navigation Marks for Met and Not-Met Sorties. The method of log analysis described in Sec.1.1 has justified itself since it has revealed the average standard of navigation within the Command, and at the same time has shown the specific points where improvement can be effected. It is now of some interest to see what relation, if any, exists between the navigation mark and the accuracy of the navigation. A simple criterion, applicable to all logs, of accuracy in navigation is provided by success or failure in meeting the convoy. In Table IV are listed for four distance ranges the number of met and not-met sorties with the average navigation mark for each; the table refers to 437 sorties, 7 sorties being rejected since in these the aircraft returned to base before reaching the convoy area. From these mean navigation marks, to which are attached the 50 per cent errors of the means, it will be seen that though there is an increase with increase in the range of the sortie, there is no significant difference between met and not-met sorties. The method of log analysis laid down in Sec.1.1 therefore fails to indicate the accuracy of the navigation.

What are the causes of this failure? One undoubtedly is the treatment of wind lane and drift as a half weight determination of wind velocity. As already pointed out (Sec.1.21) this method gives a most inaccurate determination of wind velocity, and therefore of ground speed, and is in effect nothing more than a means of maintaining track. It is, however, possible for a navigator to get full marks for such determinations while a better navigator, finding winds infrequently but accurately from multiple drifts, gets only a small fraction of the total mark. The present method of scoring would have introduced no serious difficulty had it not been that throughout the Command over 60 per cent of the winds were found by this inaccurate method; in any revision of the system of scoring it would be better to count such wind lane and drift determinations of wind velocity only as drifts, - this being in effect all they are. The other cause of failure lies in the method of marking wireless and

Table IV. Navigation Marks and Meeting.

Range	Met Sorties		Not-met Sorties	
	No.	Mark	No.	Mark
n.m.				
<200	142	3.52 ± .08	18	3.77 ± .21
200-300	116	3.76 .08	18	3.33 .18
300-400	65	3.82 .09	15	3.70 .19
>400	48	4.24 .09	15	3.96 .22

sextant aids to navigation. Since the error of D.R. navigation increases, at least for straight runs, linearly with the distance, it is essential to have a method of frequently determining ground position which is independent of the distance. This is provided to a limited extent by wireless fixes, and ideally consequently in any reasonably rapid method of analysis no discrimination can be made between used and unused fixes. These two causes seem sufficient to account for the lack of dependence of meeting on the navigation mark, and to suggest that such methods of analysis can only reveal the standard of navigation, not its accuracy.

Sec. 2. The Accuracy of Navigation.

2.1. The Method. The accuracy in which we are interested is the accuracy in the position of the aircraft at the end of its run out to the convoy area. The Final Error obtained from the landfall on the return to base, apart from the fact that it is not wholly objective, is therefore not suitable since it includes the effect of all the errors in navigation made during the patrol, as well as those made on the usually poorly navigated homeward trip; further there is a tendency for navigation to be abandoned as soon as the A.S.V. Beacon is picked up on the return journey, so that final errors are not available for many sorties otherwise suitable for discussion. For these reasons the Final Errors used in the present discussion are those found from the difference in position of the convoy as given by the aircraft navigator in his log and as given by the convoy Commodore in his log. This error, termed in what follows the Combined Error, is due to the error in aircraft navigation and to the error in convoy position as deduced from its log. The accuracy of aircraft navigation found by this method, while it has the advantage of being strictly objective and of referring to the part of the sortie in which we are interested, suffers therefore from the disadvantages that in the first instance only a Combined Error is given, and secondly that it can only be applied to those sorties for which logs of the convoy Commodore are available.

Of these disadvantages the second is more serious. Copies of the Commodore's logs were provided by the Board of Trade for convoys HG, SL, HX, SC, GF, OS, CG, AT, and WS; logs were not available for all these convoys, nor for convoys to Iceland or Russia, nor for single letter convoys, nor, most serious of all, for the numerous heavily escorted Royal Naval Forces. The consequence was that convoy positions from the Commodores' logs were available for only 105 out of the 444 sorties for which aircraft logs had been received. While this material is probably just adequate for the present discussion, it would be desirable, if future reports on this subject are required, to have available a much higher percentage of convoy logs.

2.11. The Nature of Convoy Errors. The second disadvantage of the method, namely that in the first instance only a Combined Error is obtained, can be overcome provided something is known of the error in convoy position as found from its log. This error, hereafter referred to as the Convoy Error, arises from two sources; first the error in the 0800 position of the convoy (this being the only information provided by the Board of Trade from the Commodore's log), and secondly the error in interpolating the position of the convoy at the time of meeting between the 0800 position of that day and the 0800 position of the previous or following day. The error in the 0800 position is of course wholly due to navigation error of the convoy; it arises from the error in D.R. navigation from the last astro fix, as well as from the error of the fix itself. The error of an astro position line determined from a ship at sea rarely exceeds three miles, and arises chiefly from an ill defined horizon, or from abnormal refraction to the horizon. The error of D.R. navigation arises to a small extent from errors in the Patent Log (it is normally carefully and repeatedly calibrated) and to a greater extent from uncertainty as to the speed and direction of surface currents. It has been stated (Ref.2) that it is not unusual for the error in D.R. navigation after 24 hours to vary between 10 and 20 miles. In the absence of more precise indications in the literature of sea navigation it is probably safe to assume the 50 per cent error of the 0800 convoy position to be of the order of 10 miles. Two characteristics of the error of the 0800 position should be noted. In the first place, since it depends primarily upon the error in the run from the last astro fix, and since the

That part of the Convoy Error, which is due to interpolation between two 0800 positions, arises from the assumption, necessary in the absence of other information, that the convoy travels in a straight track (rhumb line) at uniform speed between these two positions. This part of the Convoy Error may be serious and may well overwhelm the navigation error; this is well shown in the case of convoys ONS 130. On 15th September this convoy was met by Fortress S/220; the interpolated position of the convoy at the time of meeting was found to be 55°32'N, 12°21'W, while a visual signal from the convoy to the aircraft, reported in the log of the latter, gave the position as 55°15'N, 13°15'W, indicating an error due to interpolation of 35 miles.* Apart from this single instance there is no evidence available as to the size of the interpolation error. It is clear, however, that like the convoy navigation error, the interpolation error will be independent of the distance of the convoy from the British coast, and it is probably safe to assume, since the error may equally well arise from a change in course or speed of the convoy, that it is independent of the track, or shows a circular distribution.

2.2. Preliminary Examination of the Combined Errors. In order to find the Combined Error it was necessary accurately to plot the 0800 positions, and then by interpolation to find the convoy position at the time of meeting the aircraft. This plotting was carried out completely independently by each of the two writers on charts with a scale of approximately one in two million. On these same charts were also plotted the convoy position as given by the aircraft in its log, and the Combined Error was then found by a measurement of the distance and true bearing of the aircraft position of the convoy from the convoy's position of itself. In only 7 of the available 105 cases did the Combined Errors, found independently by the two writers, differ from each other by more than two nautical miles and in direction by more than 10°; these 7 cases were carefully re-examined, and an agreed Combined Error for each decided upon.

The distribution of these Combined Errors is shown in Fig.2. The origin is the interpolated position of the convoy, while the positive x and y axes are directed respectively to the east and north. The distribution of these errors appears to be roughly circular, and assuming a normal distribution in each component the probability that an error lies between x and x + dx, y and y + dy is then given by

$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)dx\,dy$$

where σ is the standard deviation, the same for both the x and the y co-ordinates. On changing to polar co-ordinates this becomes

$$\frac{1}{2\pi\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) r\,d\theta\,dr$$

where r is the amount of the error in nautical miles, and θ is the true bearing of the error. The probability that the error lies between r and r + dr is

$$F(r)\,dr = \frac{r\,dr}{2\pi\sigma^2}\int_0^{2\pi}\exp\left(-r^2/2\sigma^2\right)d\theta = \frac{1}{\sigma^2}\exp\left(-r^2/2\sigma^2\right) r\,dr \quad \ldots (1)$$

while the mean of the squares of the r's is

$$\frac{1}{\sigma^2}\int_0^\infty \exp\left(-r^2/2\sigma^2\right) r^3\,dr \Big/ \frac{1}{\sigma^2}\int_0^\infty \exp\left(-r^2/2\sigma^2\right) r\,dr = 2\sigma^2$$

or

$$\sigma^2 = \frac{1}{2}\,\Sigma(r^2)/(n-1) \quad \ldots (2)$$

where n is the number of the observations, n-1 being used in finding σ^2 to allow for one degree of freedom being used in finding the mean.

The frequency distribution of r is shown in Fig.3, while the smooth continuous curve is the normal distribution computed from eqn. (1) for a value of $\sigma = \pm 24.7$ calculated from the observed r's by eqn. (2). It will be noted

* Footnote. Probably only a part of this discrepancy is due to interpolation error. The visual signal would have been from the S.N.O. whose navigation, in view of

of $\sigma = \pm 24.7$, calculated from the observed r's by eqn.(2). It will be noted that the observed and theoretical distributions differ markedly, - the observed distribution showing an excess of both small and very large errors. Now from eqn.(1) the probability that an error lies between 0 and ρ is

$$\frac{1}{\sigma^2}\int_0^\rho \exp\left(-\frac{r^2}{2\sigma^2}\right) r\,dr = 1 - \exp\left(-\frac{\rho^2}{2\sigma^2}\right) \ldots\ldots (3)$$

From this expression and the value of σ ($= \pm 24.7$) we may find the radii of the circles such that the probability is 1/100th and 1/1000th that an error lies outside. These circles with radii 75 and 92 nautical miles respectively are shown in Fig.2, and it will be noted that four Combined Errors lie outside the 1/1000th circle.

2.21. The question of rejection of observations, particularly on some criterion based on a normal distribution of errors, is a highly debatable one, and the soundest practice favours the retention of all observations when finding means and errors. The present case is, however, somewhat different since undetected and undetectable mistakes may occur, particularly in the 0800 convoy positions since these, as received, are only typewritten copies of figures extracted from the Commodore's log. At least two instances of such mistakes have been found and allowed for in the 105 cases under consideration. It therefore seems legitimate to use a simple criterion based on the normal distribution to reject possible mistakes arising in this way, and of such criteria none is simpler than that of Wright and Hayford, which recommends the rejection of all observations for which the probability is equal to or less than 1/1000th. This criterion immediately leads to the rejection of the four Combined Errors which lie outside the 1/1000th circle in Fig.2. Following this rejection we may then use the remaining 101 observations in eqn. (2) to re-calculate σ, which now equals ± 17.9. The theoretical distribution of r from eqn.(1) using this new value of σ is shown in Fig.3 as a dotted curve. It will be noted that this new theoretical distribution fits the observed distribution very well indeed, so that it is safe to assume that the distribution of these 101 Combined Errors is very closely normal.

2.3. <u>The Combined Errors as a Function of Distance from Landfall.</u> The error in aircraft navigation may be expected to vary linearly as the distance from its last landfall to the convoy position. This follows since the flight is made on one or two legs at small angles to each other, effectively without either wireless or astro aids to navigation (Sec.1.22). The Convoy Error, however, is independent of the distance of the convoy from the British coast (Sec.2.11). Consequently an examination of Combined Errors as a function of the distance of the met convoy from the last landfall of the aircraft should permit a dissection of the Combined Error into the aircraft navigation error, which we seek, and the Convoy Error.

The 101 Combined Errors were divided according to distance flown into three roughly equal groups, - the distance flown had already been entered on the navigation analysis cards (Sec.1.1). The results of this division are shown in Table V; the upper half of this table contains in successive columns the group number, the number of sorties on which it is based, the range of the sorties in the group, the mean distance R of the sortie expressed in units of 100 nautical miles, and the mean track of the aircraft on its run out to the convoy. To arrive at the second half of the table the Combined Error, originally expressed as a distance and a bearing, is resolved into its x and y components, - x to the east and y to the north; this resolution, it will be noted from the mean track in each group, corresponds with sufficient precision to a resolution along and across the aircraft track. The errors so found are shown graphically in Fig.4, one distribution being given for each distance-group. The superposition of the errors in these three groups will lead of course to the approximately circular distribution of Fig.2, but it is of interest to note that in their dissected form the distributions are quite noticeably elliptical.

Table V. Combined Errors as Function of Distance.

Group	No. Sorties	Distances	R=Mean Dist.	Mean Track
		n.m.		
I	34	90 - 230	1.75	270°
II	34	231 - 310	2.70	269
III	33	311 - 700	4.26	261
Group	x_o	y_o	σ_x	σ_y
	n.m.	n.m.	n.m.	n.m.
I	+ 1.2	+ 3.2	± 16.0	± 13.6
II	+ 3.7	- 2.4	± 21.2	± 12.1
III	+ 1.2	- 5.0	± 24.5	± 18.3

If the distribution of errors for any distance group may be assumed to be normal, then the probability that an error lies between x and x + dx, y and y + dy is

$$\frac{1}{2\pi\sigma_x\sigma_y}\exp\left\{-\frac{(x-x_o)^2}{2\sigma_x^2}-\frac{(y-y_o)^2}{2\sigma_y^2}\right\}dx\,dy$$

where x_o, y_o are the co-ordinates of the centre of gravity of the distribution, and σ_x, σ_y (different for an elliptical distribution) are the standard deviations in the x and y directions respectively. The probability that an error lies between x and x + dx is

$$G(x)\,dx = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left\{-\frac{(x-x_o)^2}{2\sigma_x^2}\right\}dx\int_{-\infty}^{\infty}\exp\left\{-\frac{(y-y_o)^2}{2\sigma_y^2}\right\}dy = \frac{1}{\sigma_x\sqrt{2\pi}}\exp\left\{-\frac{(x-x_o)^2}{2\sigma_x^2}\right\} \quad (4)$$

and between y and y + dy is similarly

$$H(y)\,dy = \frac{1}{\sigma_y\sqrt{2\pi}}\exp\left\{-\frac{(y-y_o)^2}{2\sigma_y^2}\right\}dy \quad \ldots\ldots\ldots\ldots (5)$$

The arithmetic mean of all the x's is from (4)

$$\frac{1}{\sigma_x\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{(x-x_o)^2}{2\sigma_x^2}\right\}x\,dx \Big/ \frac{1}{\sigma_x\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{(x-x_o)^2}{2\sigma_x^2}\right\}dx = x_o$$

and similarly from (5) the arithmetic mean of the y's is y_o. The arithmetic mean of $(x-x_o)^2$ is

$$\frac{1}{\sigma_x\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{(x-x_o)^2}{2\sigma_x^2}\right\}(x-x_o)^2dx \Big/ \frac{1}{\sigma_x\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{(x-x)^2}{2\sigma_x^2}\right\}dx = \sigma_x^2,$$

and similarly of $(y-y_o)^2$ is σ_y^2

In the light of these relationships we have immediately

$$x_o = \frac{\sum x}{n} \quad ; \quad y_o = \frac{\sum y}{n} \quad \ldots\ldots (6)$$

and

$$\sigma_x^2 = \frac{\sum(x-x_o)^2}{n-1} \quad ; \quad \sigma_y^2 = \frac{\sum(y-y_o)^2}{n-1} \quad \ldots\ldots (7)$$

where n is the number of observations. The values of x_o and y_o found by an application of (6) to each group are given in the second half of Table V; it will be noted that they differ little from zero, and in view of the size of the 50 per cent errors of these means these differences are in any case not significant. Assuming $x_o = y_o = 0$, the values of the standard deviations σ_x, σ_y for each co-ordinate and each group may be found from eqn. (7). The values are given in the last two columns of the second half of the table; it will be seen that they differ for the two co-ordinates, thus confirming the elliptical distribution suggested by Fig. 4.

axes $a\sigma_x\sqrt{2}$, $a\sigma_y\sqrt{2}$, on the periphery of which the probability is constant. It is of some interest to enquire what the probability is that an error lies within this constant probability ellipse defined by the parameter a. Changing to polar co-ordinates $x = r\sin\theta$, $y = r\cos\theta$, where θ is the true bearing of the error, we find the probability that the error lies between r and r + dr, θ and $\theta + d\theta$ is given by

$$\frac{1}{2\pi\sigma_x\sigma_y} \exp\left\{-\frac{r^2}{2}\,\frac{\sigma_x^2\cos^2\theta + \sigma_y^2\sin^2\theta}{\sigma_x^2\,\sigma_y^2}\right\} r\,d\theta\,dr$$

For a given value of θ, the radius vector to the periphery of the ellipse, defined by a, is given by

$$r^2 = \frac{2a^2\sigma_x^2\,\sigma_y^2}{\sigma_x^2\cos^2\theta + \sigma_y^2\sin^2\theta}$$

Consequently the probability that the error will lie inside the a ellipse is given by

$$\left[A = \sqrt{2a^2\sigma_x^2\sigma_y^2/(\sigma_x^2\cos^2\theta + \sigma_y^2\sin^2\theta)}\right]$$

$$\frac{1}{2\pi\sigma_x\sigma_y}\int_0^{2\pi} d\theta \int_0^{A} \exp\left\{-\frac{r^2}{2}\,\frac{\sigma_x^2\cos^2\theta + \sigma_y^2\sin^2\theta}{\sigma_x^2\,\sigma_y^2}\right\} dr$$

$$= \frac{\sigma_x\sigma_y\{1-\exp(-a^2)\}}{2\pi}\int_0^{2\pi}\frac{d\theta}{\sigma_x^2\cos^2\theta + \sigma_y^2\sin^2\theta} = 1-\exp(-a^2). \quad (8)$$

Thus if we make $1 - \exp(-a^2) = 0.5$, we can find a, and so the axes of the 50 per cent ellipse. These ellipses, as well as those for which the probability is 1/100th that an error lies outside, are shown in Fig.4; it will be noted that the distribution of errors is as closely normal as would be expected from such small samples.

2.4. Determination of the Error of Aircraft Navigation. The standard deviation of the Combined Error in each component is given by

$$\sigma_x^2 = C^2 + \sigma_1^2 = C^2 + A_1^2R^2 \;:\; \sigma_y^2 = C^2 + \sigma_2^2 = C^2 + A_2^2R^2 \;\ldots\ldots\; (9)$$

C is the standard deviation of the Convoy Error, independent of distance from the British coast and the same in each component; σ_1 is the standard deviation along the track of the navigation error of the aircraft, and this, as already noted (Sec.2.3), varies linearly as the distance R (in units of 100 nautical miles) of the aircraft from its last landfall. Similarly σ_2 is the standard deviation of the aircraft error across its track. Since the Convoy Error, C, and the aircraft errors σ_1, σ_2, are independent, and since normal distributions reproduce themselves, we immediately obtain the relationships given in eqn.(9). From Table.V we have σ_x, σ_y as functions of R, and a least squares solution of the resulting six observation equations gives

$$\left.\begin{aligned} C &= \pm\, 13.2 \\ \sigma_1 &= \pm\, 5.44R \\ \sigma_2 &= \pm\, 2.65R \end{aligned}\right\}\ \text{nautical miles} \quad \ldots\ldots\ (10)$$

From eqns.(3,8) and the values found in (10), we may readily calculate the dimensions of the 50 per cent ellipses. Thus 50 per cent of the interpolated convoy positions will occur in a circle of radius 15.6 nautical miles, while 50 per cent of the aircraft errors will lie in an ellipse whose semi-axes are

$$\left.\begin{aligned} \text{Semi-axis major} &= 5.96\text{ R} \\ \text{Semi-axis minor} &= 3.12\text{ R} \end{aligned}\right\}\ \text{nautical miles} \quad \ldots\ldots\ (11)$$

It should once again be emphasized that the 50 per cent Convoy Error of 15.6 nautical miles is made up of a probably small navigational error of the convoy, and a probably large error due to interpolation between the 0800 positions.

The 50 per cent errors of aircraft navigation thus vary as six per cent of the distance run along the track and three per cent of this distance across the track. This elliptical distribution is precisely what we would have

Sec. 3. The Probability of Meeting a Convoy.

3.1 Observed Frequency of Meeting. Of the 444 sorties in the period 1 March to 15 October 1942 for which aircraft logs were available and have been marked, only 7 referred to aircraft which, for various reasons, returned to base before reaching the convoy area. For the remaining 437 sorties Table VI gives the percentage which met convoys as a function of the distance flown by the aircraft from its last landfall; the first four columns give the distance range, the total number of sorties made within this range, the mean distance to which the aircraft flew (expressed in units of 100 nautical miles), and the percentage of sorties in which the aircraft met the convoy. The remaining columns of the table refer to the search.

From the table and from Fig.5, which shows percentage met as a function of distance, it will be seen that this percentage decreases linearly. Assuming the percentage to be 100 at the coast, a least squares solution shows that the percentage met, p, is given by

$$p = 100 - (5.18 \pm 0.33) R \quad \ldots\ldots \quad (12)$$

where the factor of R has of course the dimensions of a reciprocal of a length. It is possible also to analyse this material according to the direction of motion of the convoy, as well as its distance. Of the 437 sorties 33 per cent composed mostly of Royal Naval Forces, are to convoys whose direction of motion is unknown; this leaves 203 sorties to inward convoys for which a least squares solution gives a factor of proportionality of 4.62 ± 0.26, and 88 sorties to outward convoys for which the factor of proportionality is 3.76 ± 0.50. In view of the size of the 50 per cent errors of these factors it may be concluded that the present material shows no significant difference between outward and inward convoys.

Table VI. Meeting of Convoys.

Range	Total No. Sorties	R=Mean Distance	Percentage Met	Met Sorties: Percentage Sorties with Search Duration 0^h	0.01^h-1.00^h	$>1.00^h$	Not met Sorties Mean Search Duration
n.m.							
< 200	160	1.24	88.8	57.6	31.7	10.7	$3^h.12$
200-299	134	2.48	86.6	43.1	41.4	15.5	2 .92
300-399	80	3.36	81.2	41.5	40.0	18.5	2 .40
≥ 400	63	5.05	76.2	39.6	35.4	25.0	1 .90

During the period under review the principal type of search carried out by the aircraft on reaching the convoy area was the standard Creeping Line Ahead search. In 42 of the 437 sorties, however, a search was attempted by wireless homing; in 33 of these attempts contact was established, and with one exception these contacts led to meeting the convoy. In effect then the searches referred to in Table VI are C.L.A. searches, since over 92 per cent were made by this method. For the met sorties the search duration shows a strongly marked J-shaped distribution, so that instead of the arithmetic mean, somewhat meaningless for such distributions, the percentage of sorties with no search, and with searches under and over one hour are given in Table VI. It will be noted that even at the extreme range nearly 40 per cent of the sorties met with no search at all, and that this percentage steadily increased as the distance of the convoy from land decreased. For the not-met sorties the duration of the search shows no asymmetry, and the arithmetic mean, which may be used in this case, increases from a search of nearly two hours at the extreme range to over three hours for the inner ranges.

The duration of the C.L.A. search, as well as its effectiveness, depends mutually exclusive, that is the eye visibility may be greater than 10 miles at the same time as no A.S.V. is carried. Considering all the sorties together it will be seen that A.S.V. was used for searching on 211, or 48 per cent of the possible occasions; on 144 of the occasions when it was not used, the eye visibility was greater than 10 miles, and on 67 occasions, which may have included some of these great eye visibilities, no A.S.V. was available. The limited use of A.S.V. therefore appears to be attributable to the use of the eye, when its visibility is comparable with or greater than that of A.S.V., and in part to the fact that A.S.V. was not carried on a number of aircraft. Regarding the sorties as a whole, it is clear that the eye visibility was good over the period covered by these sorties; this is clearly shown by the last four columns of Table VII which give the percentage

Table VII - Number Searches by A.S.V. and by Eye.

Sortie	A S V	Eye	Total	Eye Searches		Percentage Sorties for which Eye Vis. =			
				Vis ≥ 10	No ASV	0-2 n.m.	2-6 n.m.	6-10 n.m.	≥ 10 n.m.
						%	%	%	%
Met	166	205	371	142	62	13.8	18.7	12.7	54.8
Not-met	45	21	66	2	5	50.0	16.2	3.2	30.6
All	211	226	437	144	67	19.1	18.3	11.3	51.3

of sorties with different eye visibilities (arithmetic means cannot be used on account of the highly asymmetrical distribution of the eye visibilities). It will be noted that in over 50 per cent of the sorties the eye visibility was greater than 10 miles, and in over 60 per cent was greater than 6 miles. A curious feature of these distributions is the high percentage of visibilities under two miles for the not-met sorties. This result should not be interpreted as meaning that A.S.V. plays no useful purpose in searching, which is clearly absurd, but rather that meeting depends upon both the eye and A.S.V., first contacts by the latter having always to be converted to eye-contacts before the convoy can be regarded as met. Summarizing, it is probably not unfair to regard the visibility, using the term indifferently for either the eye or A.S.V., as lying between 10 and 15 miles for the searches listed in Table VII.

It is of some interest to compare these results with those obtained in the earlier period July to December 1941 (Ref.1). The 679 sorties available for discussion in this period show also a closely linear dependence of meeting on distance, a least squares solution leading to a factor of proportionality of 8.20 ± 0.27. Not only does this represent a distinctly smaller probability of meeting than given by eqn.(12), but also there is for this earlier period distinct evidence of a difference between inward and outward convoys. The former show apparently a non-linear dependence of meeting on distance of such a character that up to 400 miles inward convoys are less frequently met than outward.

3.2. Errors in Prediction. An aircraft on escort duty is flown to the predicted position of the convoy, and then conducts a search about that position until it meets the convoy or is forced to return to base. The observed frequency of meeting thus depends first upon the accuracy with which the aircraft is navigated to the predicted position, that is upon the aircraft navigation errors found in Sec.2, secondly upon the distance of the convoy from that position, that is upon the errors of prediction, and finally upon the area covered by the search. A determination of the errors of prediction can be made from the distance between the 0800 position of the convoy as given in the Commodore's log, and the predicted position at 0800 as given in Liverpool Forms White. The Combined Prediction Error thus found is a combination of the prediction error in which we are interested.

Of the 105 sorties discussed in Sec.2, predictions were available for 85. These predicted positions were plotted on the same charts as the convoy tracks drawn in Sec.2, and the Combined Prediction Error was measured by the vector distance from the 0800 convoy position to the 0800 Liverpool Form White position. The angle was measured counter-clockwise from the direction of motion of the convoy itself. On plotting, these 85 Combined Prediction Errors showed a highly elliptical distribution; calculation of the ellipse with a probability of 1/1000th led, using the criterion of Sec.2.21, to the rejection of two observations, one of which was identical with one of the four similarly rejected observations of Sec.2.21. The remaining 83 observations referred to 50 inward and 33 outward bound convoys. The x and y components, positive x measured along the convoy track in the direction of its motion, and positive y measured to the port beam, of the Combined Prediction Errors were then computed for these two groups. The resulting distribution of errors, the origin being the 0800 position of the convoy, is shown in Fig.6. From these components the values of x_o, y_o, and ρ_x, ρ_y, may be computed from eqns.(6) and (7). The results are given in Table VIII, where the errors attached to x_o and y_o are the 50 per cent errors of these means. It will be noted that the centroids are significantly different from zero, and since such a displacement cannot be due to convoy navigation error, there thus appears to be a tendency to predict inward convoys as being behind, and outward convoys as ahead and to starboard of their actual positions. A similar tendency, though with the opposite sign, is discernible, at least for outward convoys in the material discussed in the earlier report (Ref.1), though no comment appears to have been made on the fact.

Table VIII. Combined Prediction Error.

Convoy	n	x_o	y_o	ρ_x	ρ_y
Inward	50	-11.5 ± 3.0	$+1.0 \pm 0.9$	± 31.4	± 9.8
Outward	33	+11.2 3.2	-5.0 ± 1.4	± 26.8	± 11.8

As shown both by Fig.6 and the standard deviations in Table VIII, the distributions of the Combined Prediction Errors are highly elliptical. The axes of the 50 per cent ellipses, calculated from eqn.(8), are

	Inward	Outward	
Semi-axis major	= 37.0 n.miles	= 31.6 n.miles	along convoy track
Semi-axis minor	= 10.5	= 13.9	across convoy track

These ellipses, as well as the 99 per cent ellipses, are plotted in Fig.6; the distribution of the observations about these ellipses shows them to be as closely normal as would be expected from such small samples. The areas of the 50 per cent ellipses are 1220 square miles for inward and 1380 square miles for outward convoys. There is thus little to choose between inward and outward convoys as regards the scatter of predictions.

3.21. The Prediction Error Proper. So far the discussion has been confined to the Combined Prediction Error. If this error shows a distance effect, it would be possible to separate out the convoy navigation error, which is independent of distance, by an analysis similar to that carried out in Sec. 2.4, and so to find the prediction error proper. An analysis of the Combined Prediction Error as a function of aircraft distance of the convoy, summarized in Table IX, shows, however, that though the systematic displacement of the centroid of the distribution varies with distance, particularly for inward convoys, the scatter of errors is practically independent of distance.

Even in the absence of distance-effect, it is possible to obtain an upper

Let now c be the standard deviation, the same in all directions, of the

Table IX. Combined Prediction Error as a Function at Distance.

n	R	x_o	y_o	ρ_x	ρ_y
Inward Convoys					
8	1.50	-33.0 ± 4.8	$+6.7 \pm 1.0$	± 20.2	± 4.4
20	2.53	$-14.4 \quad 4.7$	$+3.1 \quad 4.2$	± 31.1	± 28.0
22	4.18	$-0.9 \quad 4.4$	$-2.9 \quad 1.5$	± 30.7	± 10.6
Outward Convoys					
7	1.36	$+12.2 \pm 5.4$	-8.4 ± 2.9	± 21.2	± 11.5
12	2.44	$+8.4 \quad 5.7$	$+0.4 \quad 2.3$	± 29.1	± 11.9
14	4.24	$+13.0 \quad 5.2$	$-8.5 \quad 2.0$	± 29.0	± 11.1

convoy navigation error, and let ρ_1, ρ_2 be the standard deviations, directed along and across the convoy track, of prediction error proper. Since these errors are independent we have at once

$$\rho_x^2 = c^2 + \rho_1^2 = (29.1)^2 \; ; \qquad \rho_y^2 = c^2 + \rho_2^2 = (10.8)^2$$

where the numerical values for ρ_x, ρ_y are the means for inward and outward convoys taken from Table VIII. Now since the error in the y direction for convoy navigation must be less than, or at worst, equal to, the error in prediction proper, we may put $c^2 \leqslant \rho_2^2$; since further the convoy navigation error must be greater than, or at best equal to, zero, we have $c^2 \geqslant 0$, or a lower limit to c. These inequalities may be written

$$\left.\begin{array}{l} 0 \leqslant c^2 \leqslant \rho_2^2 \\ \rho_y^2 \geqslant \rho_2^2 \geqslant \frac{1}{2}\rho_y^2 \quad \rho_x^2 \geqslant \rho_1^2 \geqslant \rho_x^2 - \frac{1}{2}\rho_y^2 \end{array}\right\} \quad \ldots\ldots (13)$$

or inserting numerical values

$$\left.\begin{array}{c} 0 \leqslant c \leqslant |\pm 7.6| \\ |\pm 29.1| \geqslant \rho_1 \geqslant |\pm 28.1| \\ |\mp 10.8| \geqslant \rho_2 \geqslant |\mp 7.6| \end{array}\right\} \quad \ldots\ldots (14)$$

The uncertainty in the prediction error proper is thus quite small, and for future discussion it will suffice to take the mean of the upper and lower limits, or

$$\left.\begin{array}{l} \rho_1 = \pm 28.6 \\ \rho_2 = \mp 9.2 \end{array} \; \text{n. miles} \right\} \quad \ldots\ldots (15)$$

3.3. Probability of Meeting a Convoy. Take the predicted position of the convoy as origin with the positive x and y axes directed respectively along the convoy track towards Gt. Britain and to the left of the track. The probability that the convoy itself lies between x and x + dx, y and y + dy is given by

$$F(x,y)\,dx\,dy = \frac{1}{2\pi\rho_1\rho_2} \exp\left\{-\frac{(x-x_o)^2}{2\rho_1^2} - \frac{(y-y_o)^2}{2\rho_2^2}\right\} dx\,dy \quad \ldots\ldots (16)$$

Here x_o and y_o are the co-ordinates of the centroid of the distribution of convoy positions, where this centroid does not lie at the origin on account of the systematic error of prediction, and ρ_1, ρ_2 are the component standard deviations of prediction error proper (eqn.15). An aircraft flies along, or at right angles to this track, to meet the convoy, and at E.T.A. predicted position has co-ordinates x', y'; the probability that it lies between x' and $x' + dx'$, y' and $y' + dy'$ is

Arrived at x', y' the aircraft carries out a search over a rectangular area $4\Delta_1\Delta_2$ from $x'-\Delta_1$ to $x'+\Delta_1$, and from $y'-\Delta_2$ to $y'+\Delta_2$. The probability that the convoy lies within this area is from (16)

$$H(x',y') = \int_{y-\Delta_2}^{y+\Delta_2}\int_{x-\Delta_1}^{x+\Delta_1} F(x,y)\,dx\,dy \qquad \ldots\ldots (18),$$

while the independent probability that the aircraft has reached this particular starting point for its search is given by (17). Hence the probability of meeting under these particular circumstances is

$$G(x',y')\,.\,H(x',y')\,dx'dy' \qquad \ldots\ldots (19)$$

However, there is no restriction on the possible starting points of the search, and the probability P of the aircraft meeting the convoy after a search over an area $\Delta_1\Delta_2$ is the sum of the mutually exclusive probabilities (19) for all possible values of x' and y', or

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(x',y')\cdot H(x',y')\,dx'dy' \qquad \ldots\ldots (20)$$

When $x_o = y_o = 0$ this integral has been evaluated by Professor E.A. Milne (see Appendix) with the result that

$$P = \operatorname{erf}\frac{\Delta_1}{\sqrt{2(\rho_1^2+\sigma_1^2)}}\;\operatorname{erf}\frac{\Delta_2}{\sqrt{2(\rho_2^2+\sigma_2^2)}} \qquad \ldots\ldots (21)$$

where

$$\operatorname{erf} z = \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)\,dt$$

is the tabulated Error Function.

In practice not only are x_o and y_o different from zero, but also the search is extended unequally on either side of the predicted position. If in (18) we change the limits of integration of the integral in x' to $x'-\delta_1$ and $x'+\Delta_1$, and of the integral in y' to $y'-\delta_2$ and $y'+\Delta_2$, and at the same time explicitly retain x_o and y_o, then the integration of (20) with the newly defined $H(x', y')$ will correspond to any required practical problem. It readily follows after one or two minor algebraic changes in Milne's derivation that

$$P_1 = \frac{1}{4}\left[\operatorname{erf}\frac{\delta_1+x_o}{\sqrt{2(\rho_1^2+\sigma_1^2)}} + \operatorname{erf}\frac{\Delta_1-x_o}{\sqrt{2(\rho_1^2+\sigma_1^2)}}\right]\left[\operatorname{erf}\frac{\delta_2+y_o}{\sqrt{2(\rho_2^2+\sigma_2^2)}} + \operatorname{erf}\frac{\Delta_2-y_o}{\sqrt{2(\rho_2^2+\sigma_2^2)}}\right] \ldots\ldots (22)$$

3.31. Probability of Meeting under Existing Conditions. There are three main cases of convoy interception. (i) The convoy and the aircraft both move in an east-westerly direction (e.g. convoys HX, SC, On, Os); (ii) The convoy and the aircraft both move in a north-southerly direction (e.g. convoys HG, SL, OG in the Bay); and (iii) the convoy moves north-south and the aircraft east-west (e.g. convoys HG, SL, ON west of Ireland). Cases (i) and (ii) as far as the evaluation of the probability of meeting is concerned are identical, and since such cases include by far the majority of convoy interceptions they alone will be treated in what follows.

Similarly though in some 8 per cent of the sorties discussed in this report the search was conducted by Wireless Homing, it will be assumed for simplicity that all the searches were carried out by the standard C.L.A. method. For inward convoys this commences 25 miles ahead of the predicted position of the convoy, and extends 15 miles on either side of the convoy track to a [illegible]

Table X. Standard Creeping Line Ahead Search.

Convoy	σ_1	Δ_1	σ_2	Δ_2	Duration
Vis. = 10 n.m.					h
Inward	50	25	25	25	1.64
Outward	35	20	25	25	1.24
Vis. = 15 n.m.					
Inward	55	25	30	30	1.44
Outward	40	20	30	30	0.96

The constants needed for the evaluation of eqn. (22) are given in Table XI for four different values of R, the distance run by the aircraft expressed in units of 100 n.miles. The values of σ_1 and σ_2 are derived immediately from eqn. (10), Sec. 2.4, the values of ρ_1 and ρ_2 are taken from eqn. (15), Sec. 3.21, and the values of x_0 and y_0 are smoothed values with the signs appropriate to the change of origin and change of axis directions taken from Table IX, Sec. 3.21. Inserting the appropriate

Table XI. Values of σ, ρ, x_0 and y_0

R	σ_1	σ_2	ρ_1	ρ_2	Inward x_0	Inward y_0	Outward x_0	Outward y_0
1.50	±8.2	± 4.0	±28.6	± 9.2	+ 20	0	+ 11	- 5
3.00	16.3	8.0	28.6	9.2	+ 20	0	+ 11	- 5
4.50	24.5	11.9	28.6	9.2	0	0	+ 11	- 5
6.00	32.6	15.9	28.6	9.2	0	0	+ 11	- 5

values from Tables X and XI in eqn. (22), we find the values of P, given in Table XII. This table is divided in two halves, the first containing the predicted P, for a visibility distance of 10 n.miles, the second that for a visibility distance of 15 n.miles; inward and outward convoys are considered separately. The last column of the Table contains for comparison the actual probability of meeting, expressed as a fraction and computed from eqn. (12) of Sec. 3.1. It will be noted that the predicted probability of meeting is not

Table XII. Predicted Probability of Meeting

Vis.	R	Inward Convoys			Outward Convoys			Observed Probability at Meeting
		P_1	P_2	$P=P_1+P_2$	P_1	P_2	$P=P_1+P_2$	
10 n.m.	1.50	0.550	0.283	0.833	0.545	0.189	0.734	0.922
	3.00	.521	.254	.775	.488	.212	.700	.845
	4.50	.591	.123	.714	.427	.185	.612	.767
	6.00	.491	.112	.603	.356	.157	.513	.690
15 n.m.	1.50	0.560	0.372	0.932	0.572	0.310	0.882	0.922
	3.00	.542	.338	.880	.535	.288	.823	.845
	4.50	.643	.170	.813	.477	.257	.734	.767
	6.00	.552	.159	.711	.409	.223	.632	.690

only considerably smaller than that observed, but also for inward convoys shows little dependence on distance. A clue to the origin of this discrepancy is and whose length is the distance run along the convoy track. Let P_2 be the probability of meeting in this approach area; then it may be shown from Sec. 3.3 that

$$P_2 = \frac{1}{4}\left[\operatorname{erf}\frac{\Delta_1+\lambda-x_0}{\sqrt{2(\rho_1^2+\sigma_1^2)}} - \operatorname{erf}\frac{\Delta_1-x_0}{\sqrt{2(\rho_1^2+\sigma_1^2)}}\right]\left[\operatorname{erf}\frac{\epsilon+y_0}{\sqrt{2(\rho_2^2+\sigma_2^2)}} + \operatorname{erf}\frac{\epsilon-y_0}{\sqrt{2(\rho_2^2+\sigma_2^2)}}\right] \quad \ldots (23)$$

where λ is the distance flown along the convoy track up to the commencement of the C.L.A. search, and ϵ is the visibility distance. λ may be taken to be 75 miles, which presupposes that the aircraft is flying along the convoy track for 100 n.miles before it reaches the predicted position of the convoy, while ϵ is either 10 or 15 miles. The resulting values of P_2 are given in Table XII, and it will be noted that there is a quite substantial probability of meeting before the commencement of the C.L.A. search. The total probability of meeting, P, is of course the sum of the mutually exclusive probabilities of meeting in the C.L.A. area, P_1, and of meeting in the approach area, P_2. The resulting values of $P = P_1 + P_2$ are given in Table XII. For inward convoys the observed probability of meeting lies between that predicted for a visibility distance of 10 miles and one of 15 miles, though considerably nearer the latter. For outward convoys the predicted probability for either visibility is systematically smaller than that observed; this is presumably a consequence of the fact that in actual practice the search for outward convoys takes as long as that for inward convoys, so that presumably the search itself extends over an area equal to that employed for inward convoys, instead of the considerably smaller area stipulated in operational orders and used for these predictions.

In short, there is good agreement between the observed probability of meeting and that predicted. This gives us a considerable measure of confidence in the accuracy of the aircraft navigation errors evaluated in Sec. 2.4, and of the prediction errors found in Sec. 3.21, and enables us now to predict with certainty the effect of changes in the mode of searching and of navigation on the probability of meeting.

3.4. <u>The Probability of Meeting for Improved Searching and Navigation.</u> Such failures to meet as still occur are clearly a consequence of errors in prediction, inadequate searching and aircraft navigational failure. With regard to errors in prediction there is probably little hope that the accidental error, with standard deviations along and across the track of 28.6 and 9.2 miles respectively, can ever be greatly reduced, so surprisingly small is it. However, it does not seem unlikely that the systematic error in prediction, amounting to some 12 or more miles, can be eliminated. In fact it is only necessary for the predictors to know of its existence not only to eliminate it, but also to discover what steps in the process of prediction are responsible for its existence. We shall therefore assume in what follows that while the accidental errors of prediction remain unchanged, the systematic errors are eliminated.

3.41. <u>Effects of an Improved Search.</u> Search along the elliptical contours of equal probability of meeting, proceeding from the predicted position outwards, would undoubtedly provide the most efficient and economical method of searching. Such a search, however, would be navigationally impracticable, and from this point of view only rectangular searches, which make the maintenance of D.R. navigation simple, need be considered. Clearly too, as long as there is no systematic error of prediction, this rectangular search must be symmetrical about the predicted position of the convoy; the probability of meeting in such a symmetrical search is given by eqn. (21). Finally the search must cover such an area that the probability of meeting is high, and yet not so high that if the search has to be completed the time consumed becomes unmanageably long; it is suggested that a probability of meeting of 0.95, that is a search designed so that only one convoy in 20 is not met, provide a suitable upper limit. The values of Δ_1 and Δ_2, such that P = 0.95, are computed from eqn. (21), and the results are given in Table XIII; the first column contains the range, the second and third the values of [illegible]

$$\operatorname{erf} \frac{\Delta_1}{\sqrt{2(\rho_1^2+\sigma_1^2)}} = \operatorname{erf} \frac{\Delta_2}{\sqrt{2(\rho_2^2+\sigma_2^2)}} = 0.975.$$

so that from tables of the error function

$$\Delta_1 = 1.585\sqrt{2(\rho_1^2+\sigma_1^2)} \quad ; \quad \Delta_2 = 1.585\sqrt{2(\rho_2^2+\sigma_2^2)}$$

The table shows how sensitive the dimensions of the constant-probability area of search are to the circumstances of meeting. Not only do the dimensions of the rectangle increase rather more rapidly than proportionately to the distance flown, but the direction of the long axis of the rectangle may change from along the convoy track to across it (with increasing distance flown), for those sorties whose track is perpendicular to that of the convoy. These changes with distance are, of course, a direct consequence of the increase in aircraft navigation-error with distance flown.

Table XIII. Search Area for P = 0.95

R	A/C Track Parallel Convoy Track				A/C Track Perpendicular Convoy Track			
	$\sqrt{2(\rho_1^2+\sigma_1^2)}$	$\sqrt{2(\rho_2^2+\sigma_2^2)}$	Δ_1	Δ_2	$\sqrt{2(\rho_1^2+\sigma_2^2)}$	$\sqrt{2(\rho_2^2+\sigma_1^2)}$	Δ_1	Δ_2
1.50	42.1	14.2	67	23	40.8	17.3	65	27
3.00	46.6	17.2	74	27	42.0	26.5	67	42
4.50	53.3	21.3	84	34	43.8	36.9	69	59
6.00	61.3	26.0	97	41	46.3	47.9	73	76
7.50	70.4	31.0	111	49	49.3	59.1	78	94
9.00	80.1	36.2	127	57	52.7	70.4	84	112

Having obtained some idea of the limiting dimensions of the rectangular area of search, the next problem is to arrange the coverage of that area in such a way as to ensure the maximum probability of meeting in the minimum time. This of course requires not only that no part of the search area be covered more than once, but also, as far as possible, that the search first be conducted in the region of the highest probability of meeting. For rectangular areas, the coverage may be either by the creeping line ahead search, or, as suggested by Group Captain Richardson, by a track search. These two methods are shown diagrammatically in Fig.7 for a rectangular area with half sides of 100 and 40 n.miles (corresponding closely with the P = 0.95 search-area at a distance of 600 miles for parallel convoy and aircraft tracks), and a visibility distance of 15 n.miles. It is now possible to calculate, by a simple extension of the methods developed in Sec.3.3, the probability of meeting the convoy at any stage in the execution of these two types of search. The results are shown graphically in Fig.8, where the probability of meeting is given as a function of the time from the commencement of the two types of search, the aircraft speed being assumed to be 125 knots, and the searches to have been conducted at 600 n.miles. From this figure it will be seen, as should be the case, that on the completion of both searches, the probability of meeting is 0.95. Actually the track search, which covers slightly more than the required area and which terminates some 20 minutes after the C.L.A. search, attains a very slightly higher probability of meeting. The interesting feature of the two curves is, however, that though tortoise-like the C.L.A. search creeps home first, for over half the time the track search is leading by a considerable amount. In this case the race is to the swift rather than to the sure, and it is clearly more economical to carry out a track search, which attains a 40 per cent probability of meeting within one hour and a 60 per cent probability within 1.75 hours, than a C.L.A. search which at corresponding times attains probabilities of meeting of only 8 and 25 per cent respectively.

It now remains only to fix provisional limits for this efficient track distance-ranges, that they will in all cases yield probabilities of meeting of 90 per cent or over. These may be compared with the probabilities of meeting now prevailing, given in the last column of the table and computed from eqn. (12). These standard searches are designed for the case where the aircraft and convoy tracks are parallel; where they are perpendicular it may be shown that A(10) or A(15) give probabilities of meeting at 150 n.miles of 0.977, and at 300 n.miles of 0.800. For greater distances, up to 600 n.miles, which is probably the practical limit at which this type of interception occurs, a square search should be carried out over a square area with half sides of 60 n.miles. This will give probabilities of meeting at 450 n.miles of P = 0.926, and at 600 n.miles of P = 0.861.

Table XIV. Probabilities of Meeting for Standard Track Searches.

R	A(10)	A(15)	B(10)	B(15)	Existing Prob.meeting
1.50	0.985	0.985			0.922
3.00	0.965	0.965			0.845
4.50	0.908	0.908	0.985	0.989	0.767
6.00			0.950	0.965	0.690
7.50			0.890	0.916	0.612
9.00			0.814	0.850	0.534

3.42. Effects of Improved Navigation. While the proposed standard Track Searches A and B will give not small probabilities of meeting in the early part of the search, the greatly improved probabilities predicted in Table XIV will only be attained if the search is, when necessary, actually completed. Assuming a speed of 125 knots the duration of the completed A(15) and B(15) searches is 2.44 and 4.92 hours respectively, and of A(10) and B(10) is 3.68 and 6.71 hours respectively. While these times are not prohibitively long and have indeed been equalled and exceeded in some of the sorties analysed in Sec.3.1, improving the method of search is a somewhat illusory method of improving the meeting of convoys; the increased time necessarily spent in a systematic and exhaustive search is only obtained at the cost of a corresponding reduction in the time spent on escort duty. A high probability of meeting, together with a large fraction of the endurance time of the aircraft available for actual escort duty, and this is what counts, can only be secured if, combined with the improved Track Search, there is a basic improvement in aircraft navigation.

A limited improvement in aircraft navigation can be immediately secured by more frequent wind finding, using multiple drifts (Secs.1.21, and 2.4). This would have the effect of making the navigation error along the track comparable with that across it; hence from eqn. (10)

$$\sigma_1 = \sigma_2 = \pm 2.65 R$$

The search area for a 0.95 probability of meeting at R = 600 n.miles would thus be reduced from $\Delta_1 = 97$, $\Delta_2 = 41$ n.miles, given in Table XIII, to 73 and 41 n.miles respectively. With sufficient accuracy this corresponds to a 25 per cent reduction in the time required for searching at this distance, and so would reduce the duration of searches B(15) and B(10) approximately to 3.7 and 5.0 hours respectively, thus saving well over an hour for useful escort duty.

A much more substantial improvement in aircraft navigation is immediately available provided full use is made of astro-fixes obtainable with the two minute integrating Mk.IXA Sextant. Exhaustive tests have shown (Ref.3) that this instrument gives a 50 per cent accidental error for a single position line of ± 2.9 n.miles together with a systematic error of ± 2.8 n.miles, the latter due to personal equation of the observer, due to errors inherent perpendicular position lines will show a constant circular distribution with $\sigma_1 = \sigma_2 = \pm 6.0$ n.miles. The search area at any distance for a 0.95 probability of meeting will thus be $\Delta_1 = 66$, $\Delta_2 = 25$ n.miles, which corresponds with sufficient accuracy to a reduction in the time of search at 600 miles to 2.1 and 2.8 hours for B(15) and B(10) respectively, that is about 50 per cent longer than the standard C.L.A. search now in use (Table X).

A still further substantial improvement is potentially available provided the sextant is slightly modified to eliminate the observer's personal equation and to reduce bubble-wander error, while at the same time a correction is made for zenith-wander. This would lead to a 50 per cent accidental error of a single position line of $\pm$ 1.6 n.miles (Ref.3), and a systematic error probably not in excess of 1.0 n.miles, giving rise on the average to a 50 per cent error of $\sqrt{(1.6)^2 + (1.0)^2} = \pm 1.9$ n.miles, or a standard deviation of $\sigma = \pm 2.8$ n.miles. This would lead to a search area, independent of distance, with dimensions for a meeting probability of P = 0.95, of $\Delta_1 = 64.5$ and $\Delta_2 = 15.2$ n.miles. Consequently track searches B(15) and B(10) could be replaced by searches taking, with sufficient accuracy, 1.2 and 1.7 hours respectively. These durations are comparable with those now taken by the existing standard C.L.A. search (cf. Table X), so that as much time would be available for useful escort duty as at present. Aircraft with improved astro-navigation would, however, have a probability of meeting of 0.95, as contrasted with the existing probability at R = 600 n.miles of 0.69 (eqn.12), or there would be an increase of efficiency for medium long range sorties of at least 38 per cent.

The substantial gain in effective escort duty which improved astro-navigation thus makes possible, will only be realized provided the limitations to the use of astro-navigation are not too crippling. These limitations are two in number. The first arises from the fact that in day sorties, when the moon is either new or full, only the sun is available for observation, and consequently only one position line, not a fix, can be obtained. The effect of this limitation can be minimized by timing some sorties so as to take advantage of darkness and the stars, and to a very substantial extent by the use of the methods described in A.P.1234 and 1456 for checking position by a single position line. Further developments in the same direction are clearly possible, as is indicated by the recent description of a new method (Ref.4), so that on occasions when only a single position line is available, skill in the use of these methods may reasonably be expected to neutralize the effect of this limitation. The second limitation arises from the impossibility of taking astro-sights in ten-tenths cloud. The question of cloud interference has already been discussed in Sec.1.23, where it is shown that there is no significant differenc in the reported cloudiness on sorties when many astro sights are taken, and sorties when none are taken. This strongly suggests that the limitation on taking of astro-sights rests with the navigator rather than with the cloud, - a suggestion amply supported by the high percentage of astro-sorties made by 120 and 201 Squadrons, and by the fact that some navigators in these two squadrons take sights on 60 to 75 per cent of their trips. Any residual limitation due to cloudiness could probably be almost completely nullified if the operational height of the aircraft on the run-out to the convoy were placed at 5000 feet or higher, that is above the top of the lower lying cloud layer (Sec.1.23), a height which would have the further desirable effect of increasing the effective range of A.S.V.

Assume that of 100 sorties with improved astro-navigation, the effect of these limitations would be to necessitate the use of D.R. navigation alone on 33 sorties, - a safe upper limit. Then the total number of mets at R = 600 n.miles (medium long range) would be $67 \times 0.95 + 33 \times 0.69 = 86.3$, where 0.95 is the probability of meeting for improved astro-navigation, and 0.69 is the probability of meeting for D.R. navigation (eqn.12). To get this same number of mets with D.R. navigation alone, as is the case at present, $100 \times 86.3 \div 69 = 125$ sorties would have to be made, or the minimum gain in efficiency from the use of improved astro-navigation would be 25 per cent.

Conclusion.

Infrequent wind finding by inadequate methods. The result of this is that in Sec.2 we find an elliptical distribution of errors in aircraft navigation, such that one half the sorties end within 6 per cent of the distance run along the track and 3 per cent of the same distance across the track. The percentage of convoys which are not met is shown in Sec.3 to be 5 per cent at 100 n.miles, and to increase linearly as the number of 100 miles run. Though a careful analysis of the probability of meeting reveals that, for short range sorties, errors in the predicted position of the convoy are primarily responsible for these failures, it is the errors of aircraft navigation which dominate the situation for medium and long range sorties. If an effective use of improved astro-navigation could be introduced, it is finally shown that the gain in actual time on escort duty would be equivalent to that obtained by increasing the number of long range aircraft, now in service with the Command and using D.R. navigation alone, by at least 25 per cent.

We must end, however, on a note of warning. It will be simple to make available improved astro-navigation, either by modifying the two-minute integrating Mk.IXA Sextant to take a reversing prism and gear, or more simply but less satisfactorily (especially for sights of the sun and moon) by adding a small telescope. The real difficulty is not then one of suitable instruments or their supply; it is in the education of navigators to take astro-sights with such instruments on every possible and many seemingly impossible occasions, and having taken the sights actually to use them to correct their D.R. navigation.

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16.4.43.

APPENDIX. SEARCH FOR CONVOY BY AIRCRAFT.

by Professor E.A.Milne, Ordnance Board.

Let (x,y) be any position of the convoy with respect to the given axes. Then the probability that the convoy lies inside the elementary domain whose opposite corners are (x,y) and (x + dx, y + dy) is given to be

$$F(x,y)\,dx\,dy \equiv \frac{1}{2\pi\rho_1\rho_2}\exp\left\{-\frac{1}{2}\left(\frac{x^2}{\rho_1^2}+\frac{y^2}{\rho_2^2}\right)\right\} \qquad (1)$$

Let (x , y) be any position of the aircraft after attempting to fly to the origin (predicted position of the convoy). Then the probability that the aircraft lies inside the elementary domain whose opposite corners are (x′ , y′) and (x′ + dx′ , y′ + dy′) is given to be

$$G(x',y')\,dx'\,dy' \equiv \frac{1}{2\pi\sigma_1\sigma_2}\exp\left\{-\frac{1}{2}\left(\frac{x'^2}{\sigma_1^2}+\frac{y'^2}{\sigma_2^2}\right)\right\} \qquad (2)$$

Now consider an aircraft which has actually flown to (x′, y′). The probability that the convoy is within $\pm\Delta_1$ of the aircraft in the x-direction and within $\pm\Delta_2$ of the aircraft in the y-direction is then

$$H(x',y') \equiv \int_{x=x'-\Delta_1}^{x=x'+\Delta_1}\int_{y=y'-\Delta_2}^{y=y'+\Delta_2} F(x,y)\,dx\,dy. \qquad (3)$$

Since the probability that the aircraft has thus flown is given by (2), the probability P that the convoy is found by a search over the region $\pm\Delta_1, \pm\Delta_2$, surrounding the aircraft's point of arrival is, for all positions of arrival of the aircraft

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(x',y')\,H(x',y')\,dx'dy' \qquad (4)$$

This is Professor H.H.Plaskett's expression.

Written out in detail this expression is

$$\left[\frac{1}{2\pi\rho_1\sigma_1}\int_{x'=-\infty}^{x'=+\infty}\exp\left(-\frac{1}{2}\frac{x'^2}{\sigma_1^2}\right)dx'\int_{x=x'-\Delta_1}^{x=x'+\Delta_1}\exp\left(-\frac{1}{2}\frac{x^2}{\rho_1^2}\right)dx\right]\left[\frac{1}{2\pi\rho_2\sigma_2}\int_{y'=-\infty}^{y'=+\infty}\exp\left(-\frac{1}{2}\frac{y'^2}{\sigma_2^2}\right)dy'\int_{y=y'-\Delta_2}^{y=y'+\Delta_2}\exp\left(-\frac{1}{2}\frac{y^2}{\rho_2^2}\right)dy\right]$$

The factor involving x′ and x, and that involving y′ and y, can be considered independently, for P may be written as

$$P = \frac{1}{(2\pi)^2\rho_1\rho_2\sigma_1\sigma_2}\int_{x'=-\infty}^{x'=+\infty}\int_{y'=-\infty}^{y'=+\infty}\exp\left\{-\frac{1}{2}\left(\frac{x'^2}{\sigma_1^2}+\frac{y'^2}{\sigma_2^2}\right)\right\}dx'dy'\int_{x=x'-\Delta_1}^{x=x'+\Delta_1}\int_{y=y'-\Delta_2}^{y=y'+\Delta_2}\exp\left\{-\frac{1}{2}\left(\frac{x^2}{\rho_1^2}+\frac{y^2}{\rho_2^2}\right)\right\}$$

Call these factors p_1 and p_2. In p_1 put

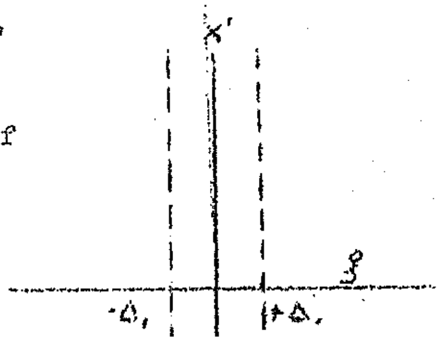
$$x = x' + \xi$$

in the inner integration. We get

$$p_1 = \frac{1}{2\pi\rho_1\sigma_1}\int_{x'=-\infty}^{x'=+\infty}\exp\left(-\frac{1}{2}\frac{x'^2}{\sigma_1^2}\right)dx'\int_{\xi=-\Delta_1}^{\xi=+\Delta_1}\exp\left(\frac{1}{2}\frac{(x'+\xi)^2}{\rho_1^2}\right)d\xi$$

This may be considered as the double integral of

$$\exp\left[-\frac{1}{2}\left\{\frac{x'^2}{\sigma_1^2}+\frac{(x'+\xi)^2}{\rho_1^2}\right\}\right]$$



taken over the strip of infinite length and breadth $2\Delta_1$ parallel to the x′ -axis.

$$p_1 = \frac{1}{2\pi\rho_1\sigma_1}\int_{\xi=-\Delta_1}^{\xi=+\Delta_1} d\xi \int_{x'=-\infty}^{x'=+\infty} \exp\left[-\frac{1}{2}\left\{\frac{x'^2}{\sigma_1^2}+\frac{(x'+\xi)^2}{\rho_1^2}\right\}\right]dx'$$

The integrand may now be re-arranged as

$$\exp\left[-\frac{1}{2}\,\frac{\sigma_1^2+\rho_1^2}{\sigma_1^2\rho_1^2}\left(x'+\xi\frac{\sigma_1^2}{\sigma_1^2+\rho_1^2}\right)^2-\frac{1}{2}\xi^2\left(\frac{1}{\rho_1^2}-\frac{\sigma_1^2}{\rho_1^2(\sigma_1^2+\rho_1^2)}\right)\right]$$

or

$$\exp\left[-\frac{1}{2}\,\frac{\sigma_1^2+\rho_1^2}{\sigma_1^2\rho_1^2}\left(x'+\xi\frac{\sigma_1^2}{\sigma_1^2+\rho_1^2}\right)^2-\frac{1}{2}\,\frac{\xi^2}{\sigma_1^2+\rho_1^2}\right]$$

Since

$$\int_{-\infty}^{\infty}\frac{\exp\left(-\frac{1}{2}\frac{\xi^2}{c^2}\right)}{c\sqrt{2\pi}} = 1$$

we have

$$p_1 = \frac{1}{2\pi\rho_1\sigma_1}\sqrt{2\pi}\,\frac{\sigma_1\rho_1}{\sqrt{\rho_1^2+\sigma_1^2}}\int_{-\Delta_1}^{\Delta_1}\exp\left[-\frac{1}{2}\,\frac{\xi^2}{\sigma_1^2+\rho_1^2}\right]d\xi$$

Now the tabulated Error Function is defined by

$$\operatorname{erf} x = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)\,dt$$

With this definition erfx tends to 1 as x tends to ∞. Putting

$$t = \xi/\sqrt{2(\rho_1^2+\sigma_1^2)}$$

in p_1, we get

$$p_1 = \frac{1}{\sqrt{2\pi}}\,\frac{1}{\sqrt{\rho_1^2+\sigma_1^2}}\sqrt{2(\rho_1^2+\sigma_1^2)}\cdot 2\int_0^{\Delta_1/\sqrt{2(\rho_1^2+\sigma_1^2)}}\exp(-t^2)\,dt = \operatorname{erf}\frac{\Delta_1}{\sqrt{2(\rho_1^2+\sigma_1^2)}}$$

The desired probability is therefore

$$P = \operatorname{erf}\frac{\Delta_1}{\sqrt{2(\rho_1^2+\sigma_1^2)}}\cdot\operatorname{erf}\frac{\Delta_2}{\sqrt{2(\rho_2^2+\sigma_2^2)}}$$

Ordnance Board.
22.2.43.

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No.	Title	Author	Reference
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2	Sea and Air Navigation	W.M.Smart	Halley Lect.1941
3	Personal Equation in Sextant Observations & its Effect on the Accuracy of Astro-navigation	W.S.Jenkins & H.H.Plaskett	A.&A.E.E./Res/172
4	Coastal Command Navigation Review		No.5

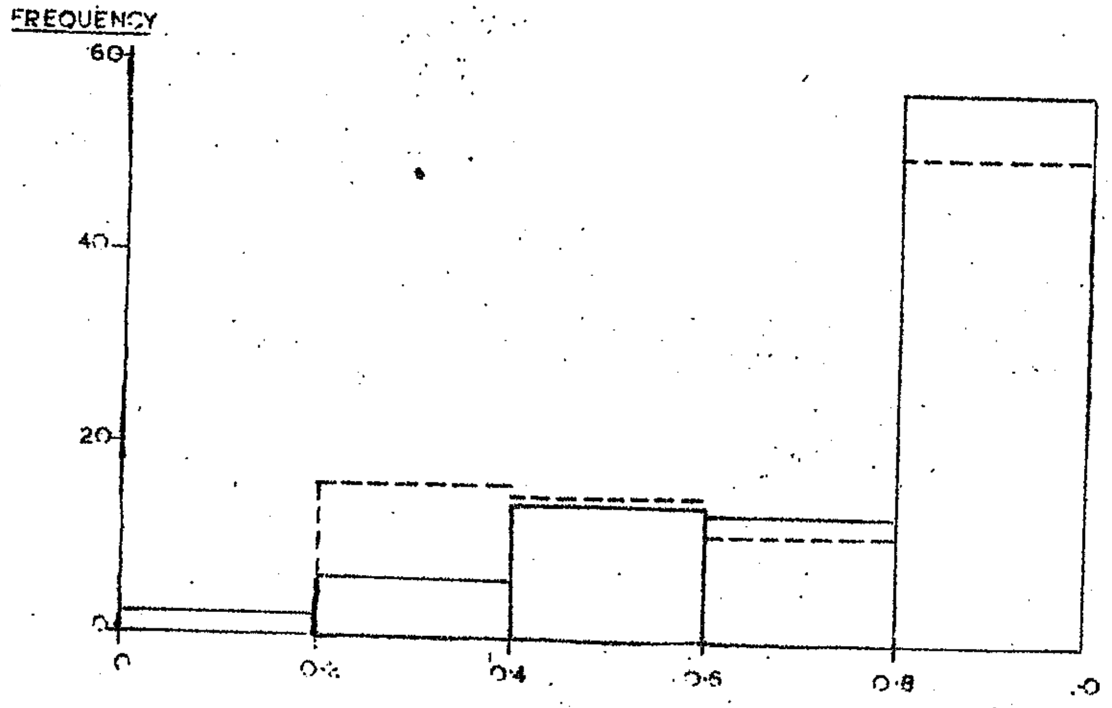


FIG: 1.

Distribution of Reported Fractions of Cloudiness, Dotted lines Astro-Sorties, full lines non-Astro Sorties. Ordinates of latter reduced in ratio 93/337 to make areas under two distributions equal.

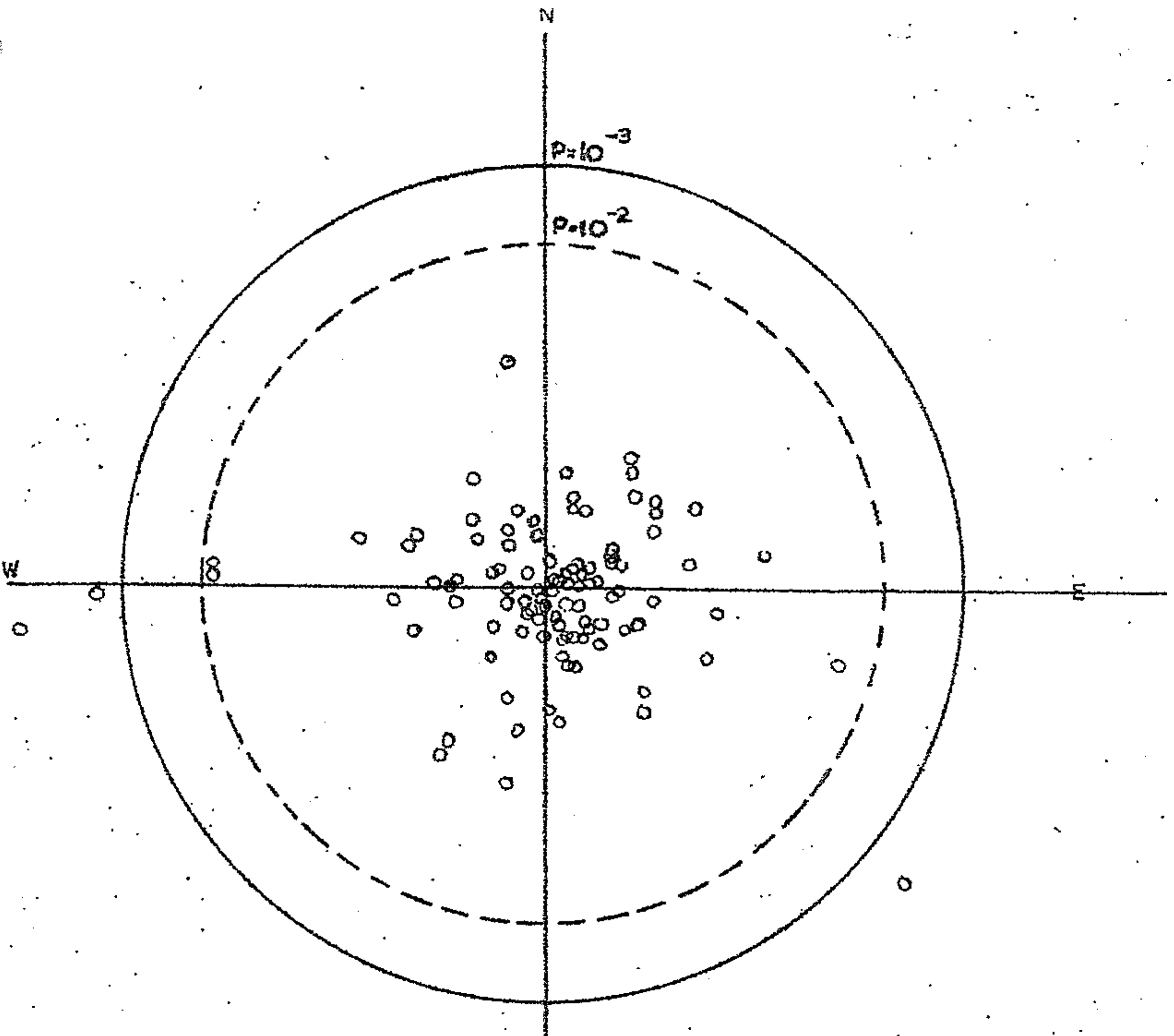


FIG: 2.
PLOT OF 105 ERRORS ON SCALE OF 40 NAUTICAL MILES = 1 INCH.

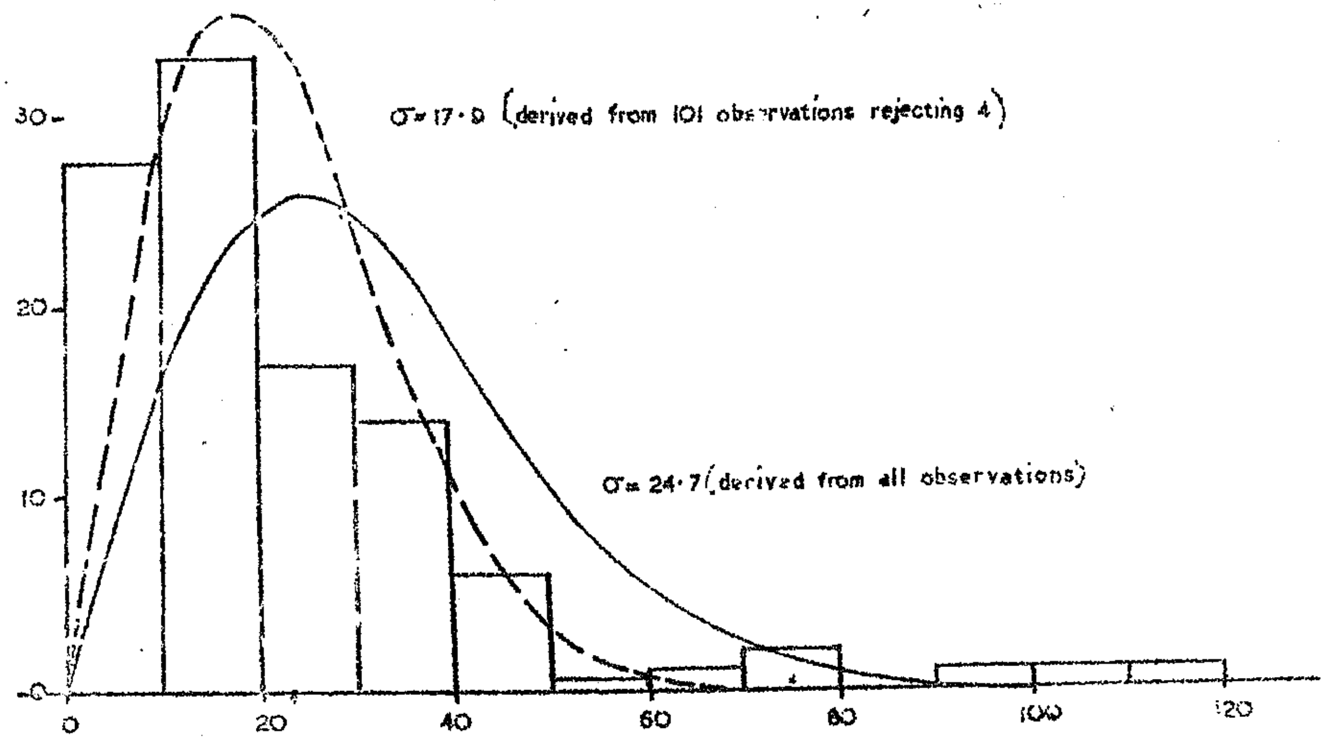


FIG 3. FREQUENCY DISTRIBUTION OF V' THE MAGNITUDE OF COMBINED ERROR

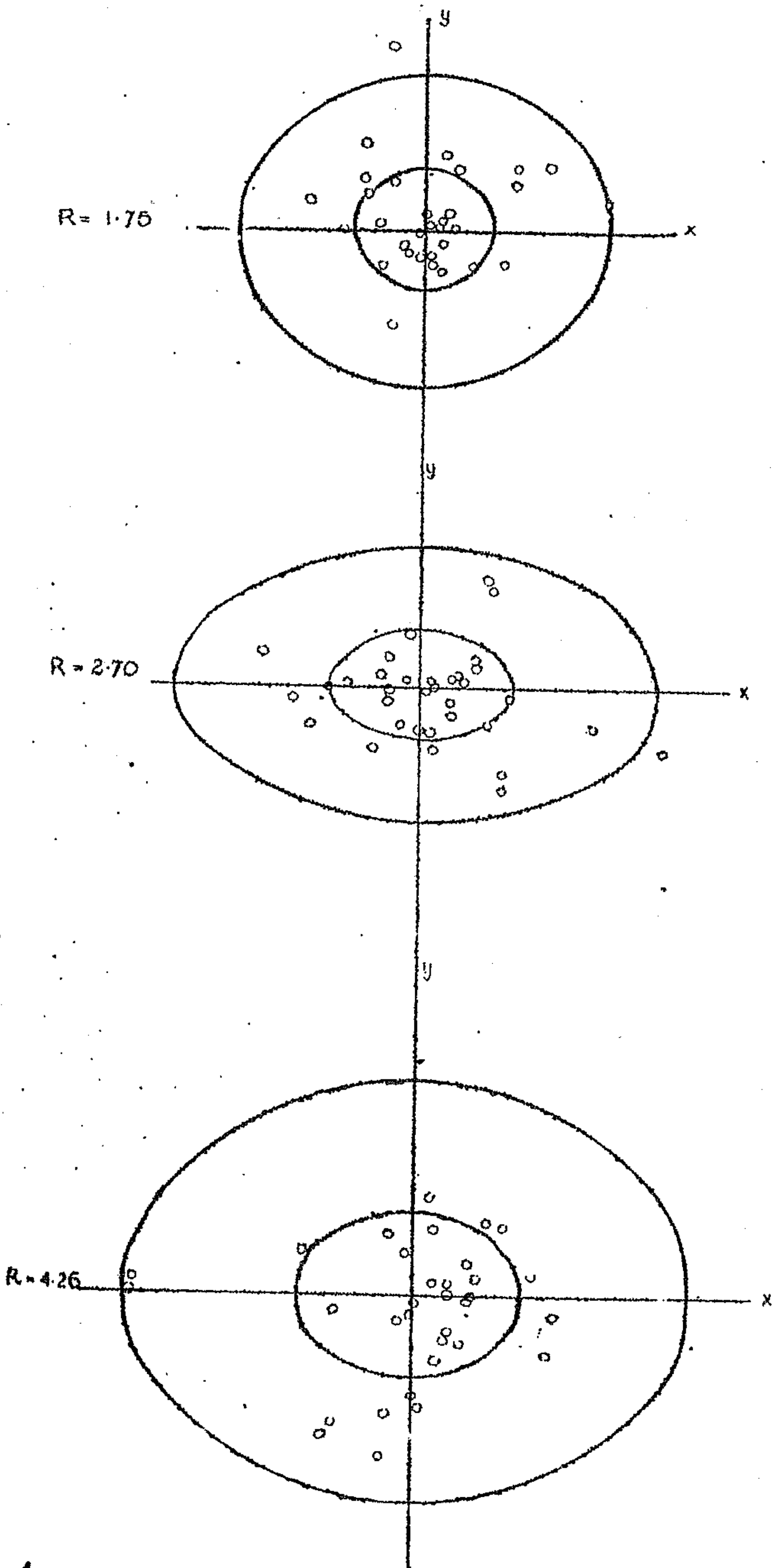


FIG. 4.
Distribution of Combined Errors for three Distance Groups.

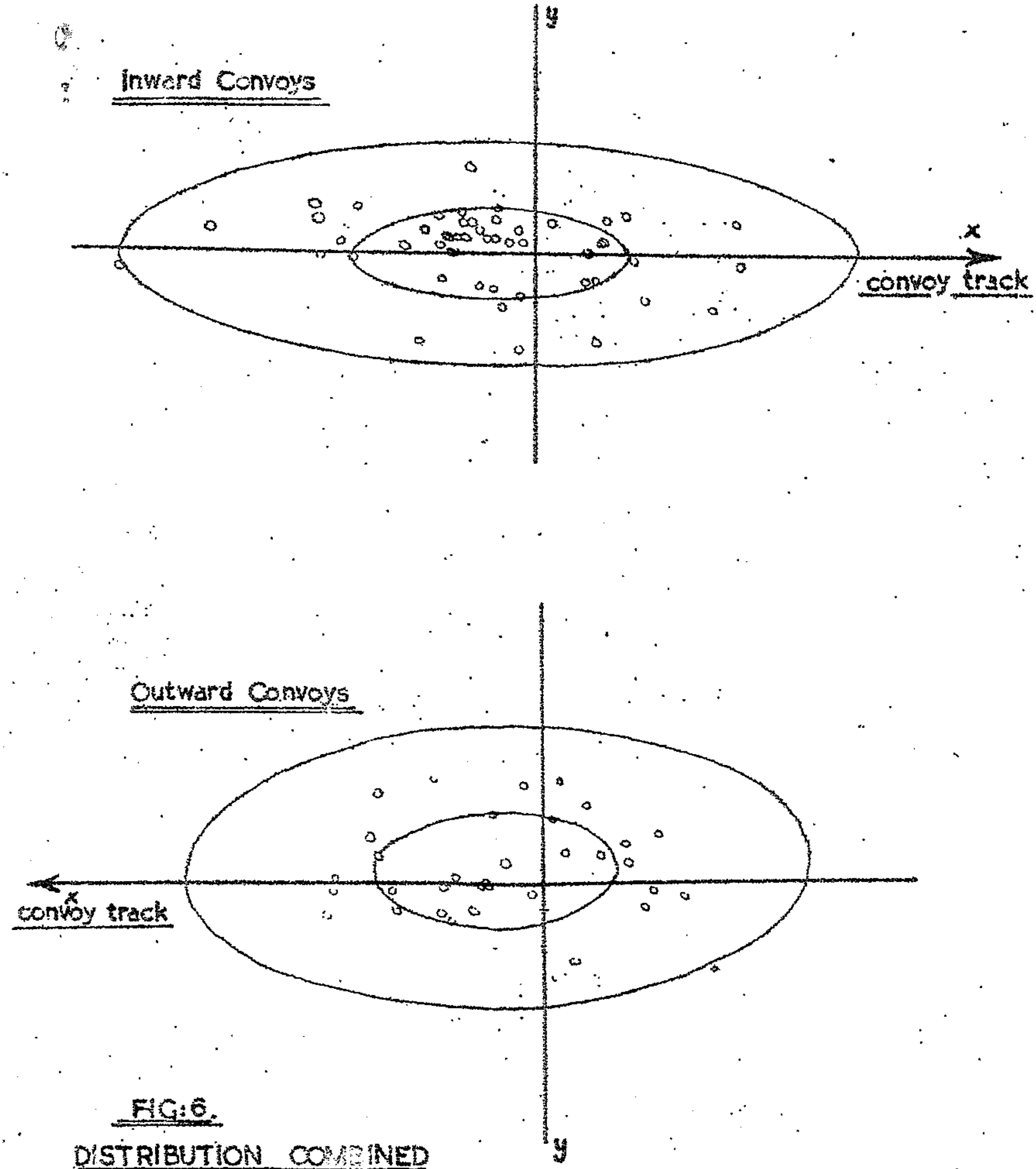


FIG: 6.

DISTRIBUTION COMBINED PREDICTION ERROR. Scale 40 n-miles. = 1 inch.

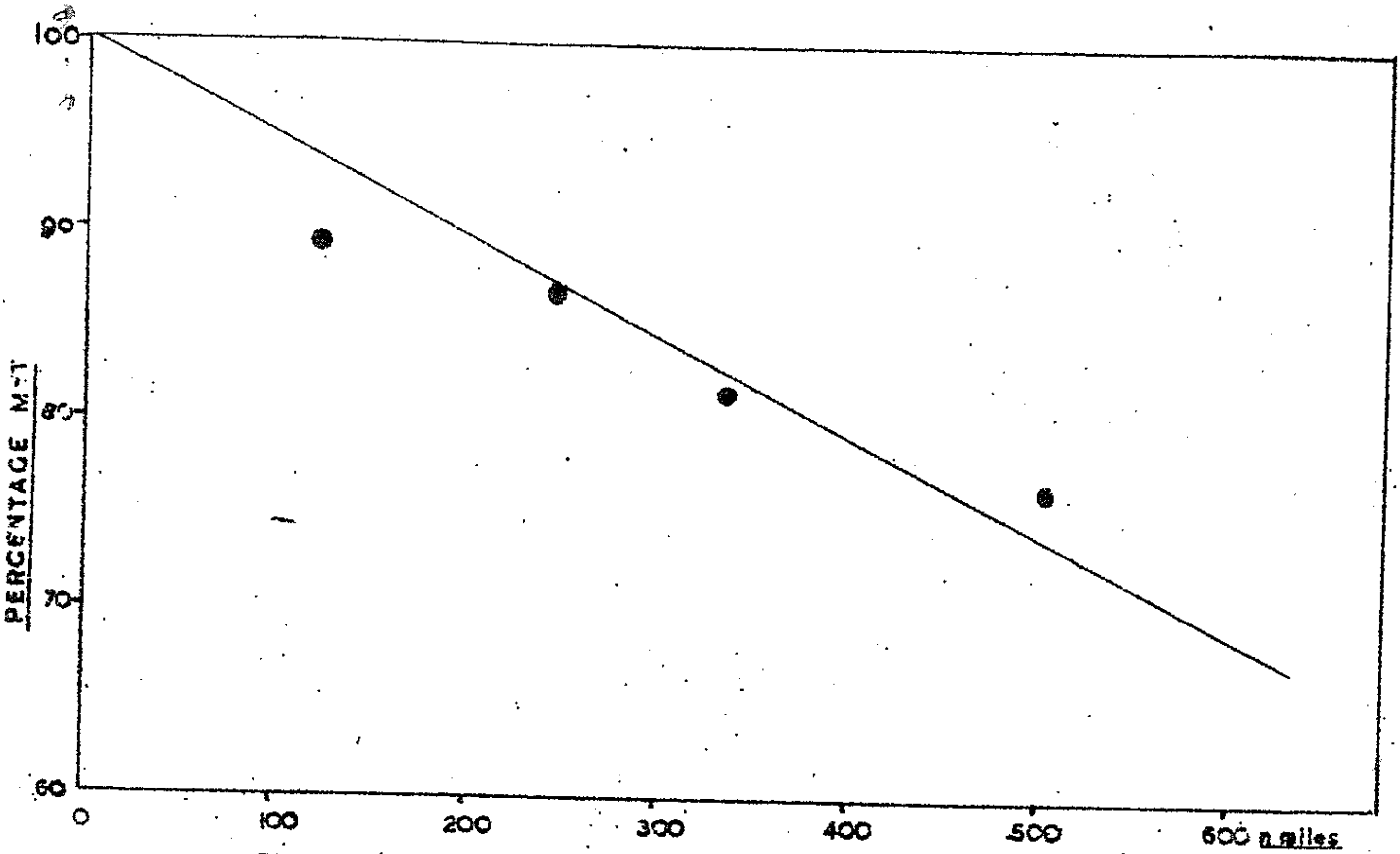


FIG 5

Percentage met as Function of Distance flown

HQ CC 444

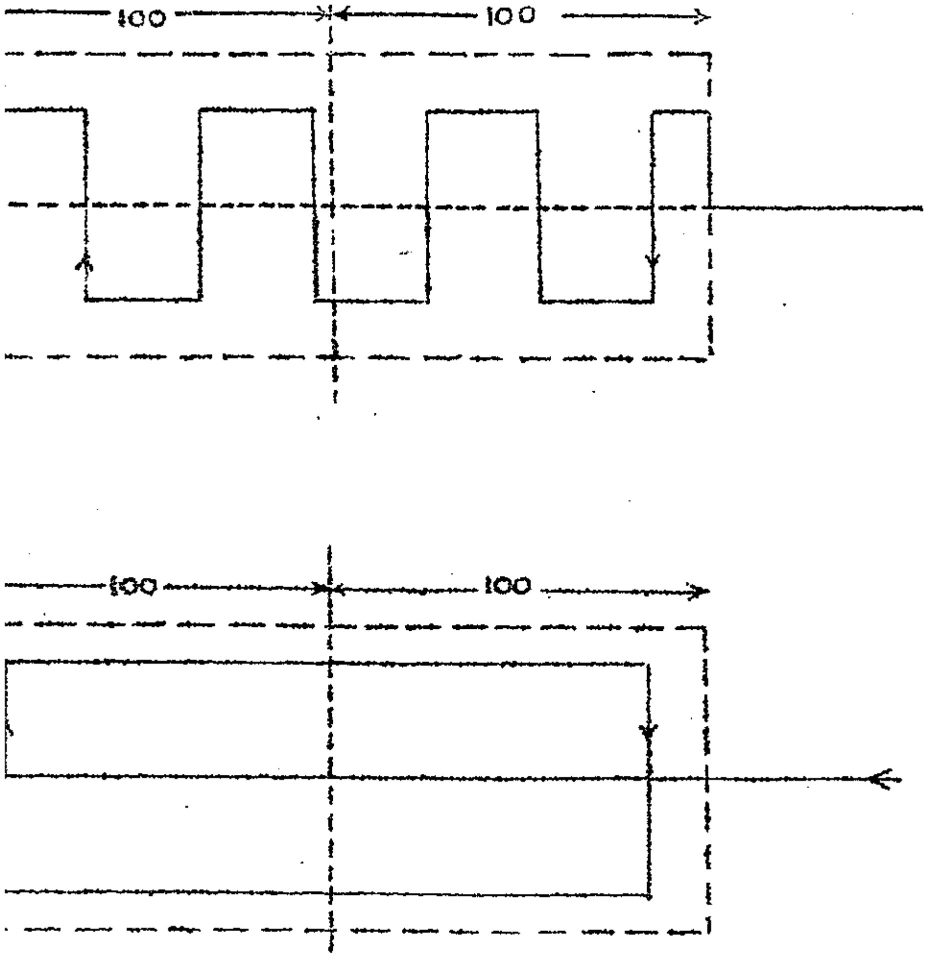


FIG. 7 C.L.A. SEARCH (upper figure) and TRACK SEARCH (lower figure) for P= 0·95 at 600 n.miles and VISIBILITY – 15 n.miles.

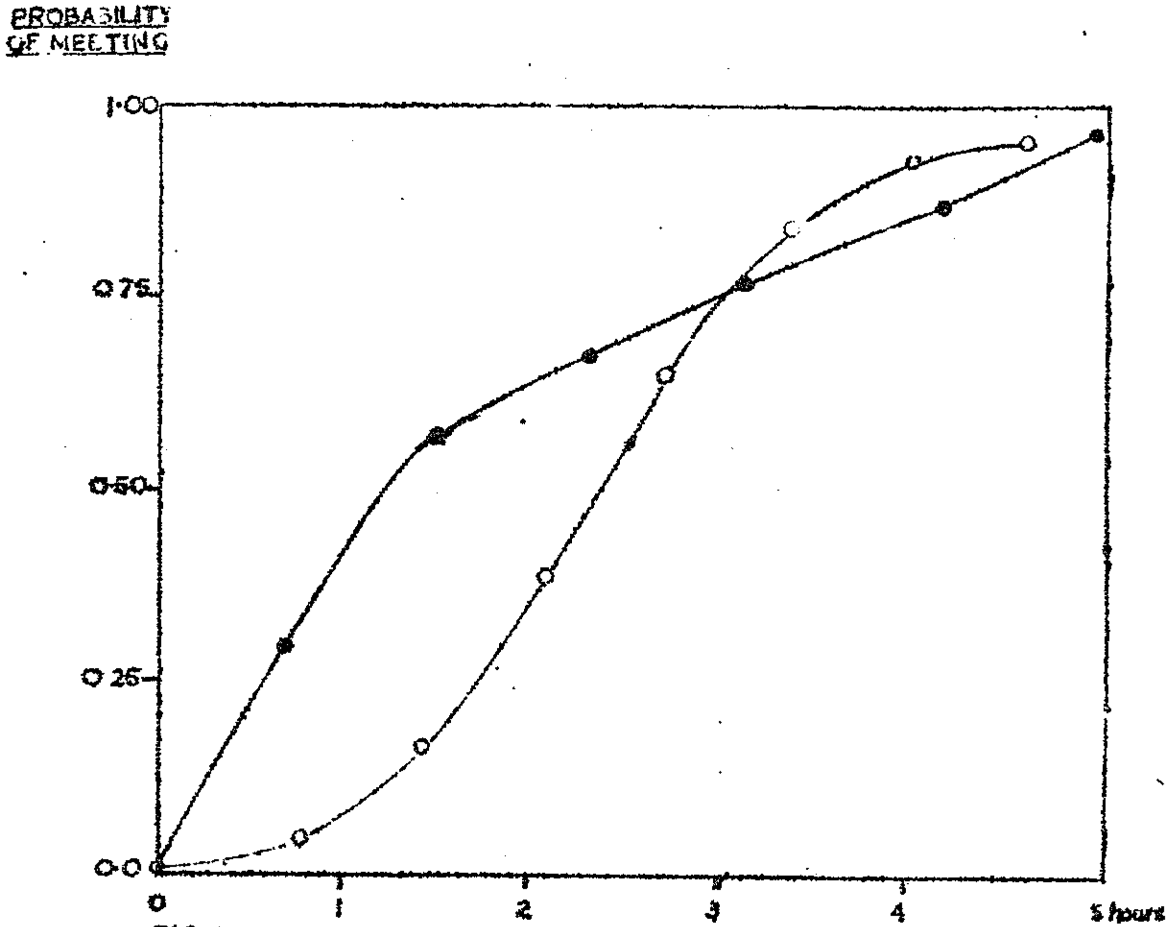


FIG. 8. PROBABILITY of MEETING as a FUNCTION OF LENGTH OF TIME SPENT ON C.L.A. SEARCH (open circles) AND ON TRACK SEARCH (filled circles). FIGURE REFERS TO THE TWO SEARCHES SKETCHED IN FIG 7.

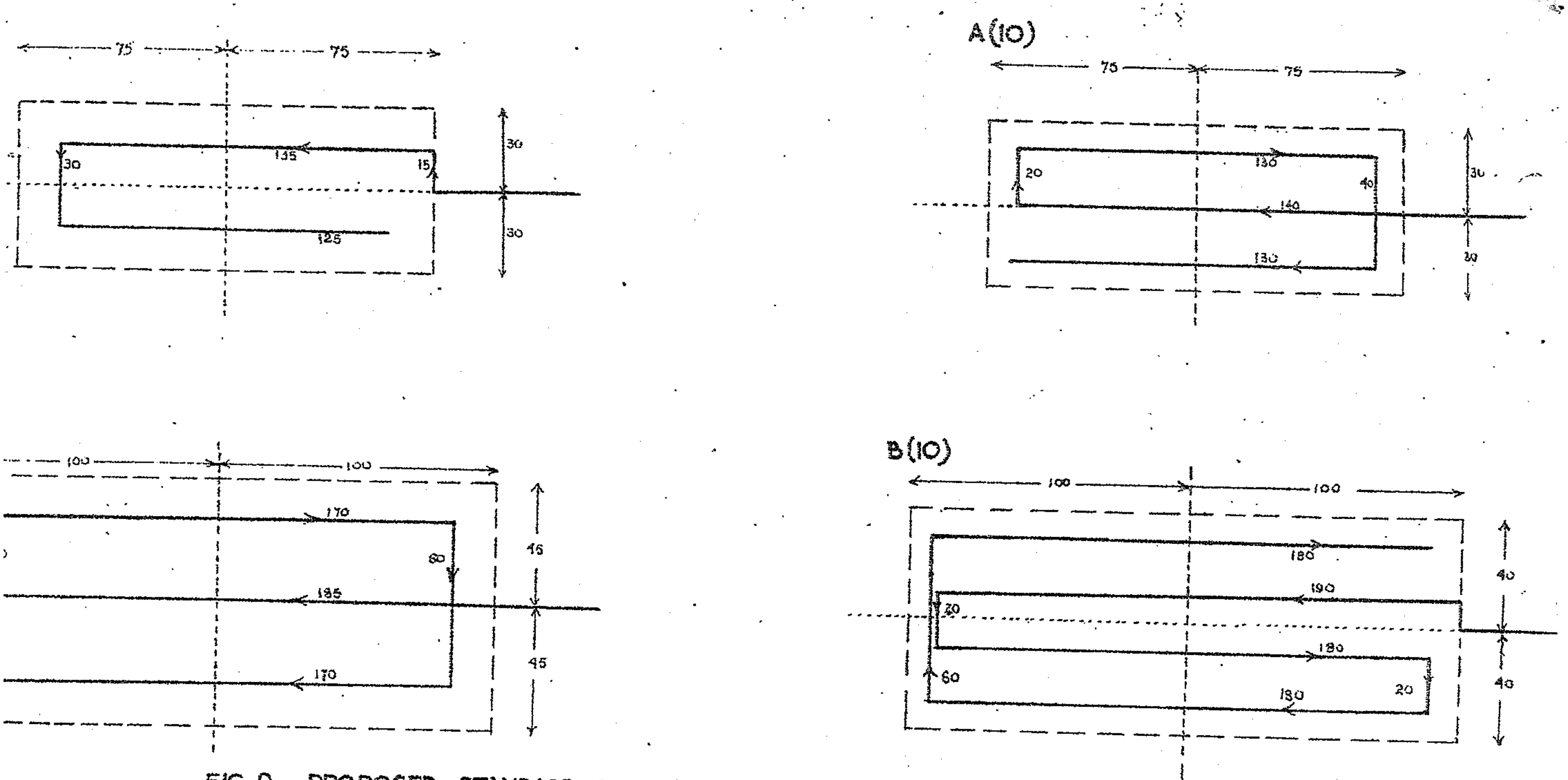


FIG 9 PROPOSED STANDARD TRACK SEARCHES FOR VISIBILITIES

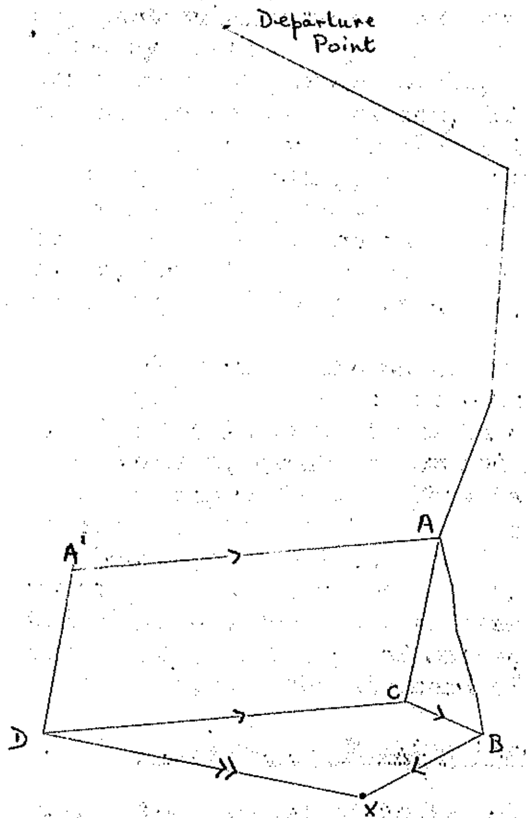
15 n.miles (left) and 10 n.miles. (right)

ANALYSIS OF NAVIGATION LOGS.

To maintain a high standard of navigation within the Command, it is essential that the navigation in each long range sortie should be carefully assessed. Heretofore it has been the practice to carry out this assessment by means of the method of analysis laid down in A.P. 1456. While theoretically sound this method suffers from the grave disadvantages of being laborious and impracticable, particularly under operational conditions. Further it is applicable only to good navigation logs, and if mean results are taken from such analyses, these means tend to give an unrepresentative and too favourably biased a view of the standard of navigation within the Command. There is therefore clearly a need to consider other methods of assessing navigation logs, and it is the purpose of this report, after a brief consideration of the standard method of analysis, to describe one such method.

1. <u>Standard Method of Analysis.</u>

1.1. The method of analysis laid down in A.P. 1456 (Chap.XXX) is as follows:- Let X be the terminal point which the aircraft is required to reach, and D be the actual ground position of the aircraft at E.T.A. terminal point. Then the Final Error of the navigation is the vector distance DX, and the purpose of the analysis is to break this Final Error into its components. From the departure point the courses steered and the distances flown, as <u>taken from the log</u>, are accurately plotted up to point A, reached at E.T.A. terminal point. Note that A is not the air plot position corresponding to the ground position D reached at this time, for each of the legs which have been plotted leading up to A are subject to what may be called Pilot's Error, that is due to the failure of the pilot to maintain the courses and speeds specified by the navigator. To find this error lay off from D the true wind vectors reversed (shown in the figure as the straight line DA), corresponding to the wind velocities which held throughout the area at the time the flight was made. The point A thus reached must therefore be the air plot position corresponding to the ground position D, and the vector A'A must be the Pilot's Error.



Now lay off from A both the wind velocities used by the navigator (shown by the broken line AB), and the true wind velocities AC, where AC is equal and parallel to A D. Consequently DC (called in A.P.1456 the Other Error) = A'A = Pilot's Error. The point B is the ground position which the navigator would have reached if his winds had been correct and if there had been no Pilot's Error. As, however, he wished to reach the terminal point X, the distance BX is the navigator's Calculation Error due to incorrect plotting, and calculation of courses to steer, while the vector CB is the error in the navigator's determination of wind (called in A.P. 1456 Wind Change Error). Consequently the Final Error DX = Vector Sum of DC (Pilot's Error), CB (Wind Error) and BX (Calculation Error), the sense of each vector being in the direction actual or true position to required position.

[illegible]

times over the route flown. The first of these can normally only be obtained from the landfall on the return to base, and this is only of use if the navigation has been continuously maintained throughout the whole sortie, including the critical period when the aircraft is homing on the A.S.V. beacon. Under operational conditions and over the sea a knowledge of the actual winds prevailing over the route is in effect impossible to obtain, particularly in view of the variability of the wind with time and place, which radio sonde ascents have revealed. Even that dubious approximation to the actual winds, namely the mean of the winds found by three or more aircraft flying over the same route at the same time, is rarely available under operation conditions.

1.2. The analysis of the Final Error into its components, as laid down in A.P. 1456, is therefore theoretically correct, and can be effected, provided the true winds and the true ground position are known. A moment's consideration, however, shows that the component errors resulting from the analysis are little more informative than the Final Error itself. Thus the Pilot's Error arises not only from the failure of the pilot to maintain the required courses and speed but also from instrument errors in the compass and A.S.I. which may not be accurately known, and / or not accurately applied by the navigator. In other words the Pilot's Error arises to a greater or less extent from failures of the navigator both on the ground and in the air. Similarly the Wind Error cannot be uniquely assigned to any particular part of the navigator's operations; thus winds determined by track and ground speed (or air plot) involve the Pilot's Error over the particular legs flown, as well as the errors of fixing the ground position, including errors of calculation, while multiple drift winds involve the accuracy of drift determinations, accuracy of piloting, and accuracy of calculation. Only the Calculation Error is unique, in that it does indicate what part of the Final Error is due to inaccurate plotting and calculation of courses to steer, but it only represents a small part of the computational errors which may be made by the navigator in the course of the flight.

In view of the uninformative nature of the analysis, the rarity with which the data can be obtained for accurately carrying it through and finally in view of its laboriousness, there seems little justification for squadron, group or command navigation officers continuing to analyse logs by this method. This is not to say that the analysis is without its use, but that use if for the individual navigator who has carried out the sortie. If he were required, following the sortie, to make the correct air plot and to find his so called Calculation Error, such an exercise, particularly if carried out under the supervision of his squadron navigation officer, could scarcely fail to improve the standard of this part of his work.

2. <u>A System for Marking Logs.</u>

2.1. The purpose of analyzing is to keep check on the standard of navigation within the Command or Group, and to discover means whereby that standard may be improved. A posible method of analysis would be to find the component errors introduced in to the Final Error as a result of each operation performed by the navigator. This has been unsuccessfully attempted in A.P. 1456, but this does not mean that it is impossible. To do it, however, would require special equipment and special personnel to be carried on simple operational flights throughout the Command, as well as a somewhat elaborate analysis of the results of such flights. Quite apart from the fact that such flights would ipso facto cease to be typical of the average operational flight, the amount of material gained in this way would perhaps be scarcely commensurate with the time and labour involved.

There is, however, another way of tackling this problem, When large, [illegible] of operational flights are being made a relatively crude statistical

ample information as to the standard of navigation and the methods whereby it can be improved. Such a method consists in selecting from amongst the various operations performed by the navigator those for which evidence of performance can be inferred from the log. If the performance or non-performance of each operation is marked on some arbitrary scale, the sum of the marks so obtained is a measure of the standard of navigation, while if success or failure of the navigation is correlated with the marks given to each operation then some conclusions as to the value of that particular operation may readily be drawn.

2.2. One such system of marking is now being applied to all long range sorties to convoys for the six month period ending 31st August 1942. While the marking is not yet complete (there is a number of logs still to be received) and no analysis of the results has been made, it is perhaps worth while at this stage to indicate the method of marking and the kind of results which it may be expected to yield.

The perfect navigation log receives a score of 10 marks. These are assigned as follows:-

1 mark for drift determinations made at the rate of 4 per hour throughout the navigational period.

3 marks for wind velocity determinations made at the rate of 2 per hours. Wind velocities estimated from wind lanes and drifts are counted as half a determintion

1 mark for accuracy of wind computation checked from data given in log on a Navigational Computer Mk.III The full mark is given if the wind given by the navigator agrees with the re-computation within 5 m.p.h. and ± 10°.

1 mark for evidence and accuracy of changing from indicated to true air speed.

1 mark for evidence of the use of the astro compass to check pilot's compass.

2 marks for obtaining one astro position line per hour.

1 mark for obtaining one loop bearing (or M.F./D.F. fix per hour).

These marks are entered on a card, one space for each mark and one card for a log. In addition the card carries the squadron number, the aircraft number and the names of the navigator and pilot; there are spaces for the day and times of flight, the distance from the last landfall to the predicted position of the convoy, a set of spaces for marking the search, and a summary of the meteorological conditions.

This method of marking has proved to be both objective and rapid. Thus a quarter of the logs have been marked independently by two computors, one with some practical experience of navigation and the other with none. In each case, apart from mistakes, the results have been in complete agreement. With regard to rapidity it is found that in so far as the required information can be obtained from the log itself, 15 minutes is sufficient to complete a card. Information which can only be obtained from Forms Orange, Group Narratives and from Convoy positions, and which is required for the analysis subsequently to be carried out, is not so easily or quickly obtained, and accounts for much of the delay experienced in completing the marking.

2.3. In all 318 logs have been marked, and the frequency distribution of the marks so obtained is shown in Table I (all tables appear at the end of the report). These marks only refer to the navigat ion carried the end of the [illegible]

The mean mark for the 317 logs is 3.6 and the standard deviation of the distribution is ± 1.3. Since 7 of the possible 10 marks are given for a not unduly high standard of D.R. navigation, it will probably be agreed that some improvement in navigation throughout the Command is still attainable. There have in fact been changes throughout the period under review. Thus in Table II will be found the mean marks for each of the six months from March to August, and from this it can bee seen that the standard steadily increased from 2.7 in March to 4.3 in May, and then tapered off to a mean of about 3.6. Similar changes are shown in the navigation of ech of the six squadrons which did the bulk of this flying.

Not only does the mean mark vary with the time, but it also varies from squadron to squadron. The results are given in Table III for all squadrons taking part in seven or more sorties, excluding the Hudson squadrons. The first column of the table contains the squadron number, the second the number of sorties made, the third the mean mark for these sorties, and the final column the standard deviation of this mean. From this table we may readily take the differences between the mean mark obtained by 120 squadron and each of the other squadrons, as well computing the standard deviation of that difference. The results are as follows:-

120 Sqdn.	- 10 Sqdn.	= +0.7	± 0.29
120	- 160	-1.2	± .41
120	201	+0.4	± .19
120	220	+0.7	± .21
120	228	+0.6	± .24

The differences are in each case at least twice as large as the standard deviation, and may therefore be regarded as significant. There is therefore a real difference between the standard of navigation maintained in the various squadrons, and the standard of navigation within the whole Command would be greatly improved if each squadron could reproduce the results obtained by 160 Squadron. It is perhaps woth noting that it and 120 Squadron, the two highest scorers, are equipped with Liberators.

Finally it is of some interest to compare the navigation marks obtained for aircraft which successfully met and those which failed to meet convoys. In Table IV are given for various ranges the total number of sorties made, the percentage of "not-mets", and the navigation marks and visibility for the mets and not-mets. The results in this table confirm results earlier obtained in ORS/CC Report No.182, namely the increase in the percentage of not-mets with increasing distance from land, and the apparent dependance of meeting on visibility. However the numbers involved in Table V are so small, that means for the not-mets have little significance. Too much stress should not therefore be laid on the fact that the percentage of not-mets increases much more slowly with distance than found in the previous work, nor are the differences of visibility for meeting and not-meeting highly significant. With similar caution we note that there seems to be little correlation between the standard of navigation as given by the present system of marking, and failure to meet. The percentage increase of not-mets with distance is undoubtedly to be attributed to the Final Error of the navigation, and its probable increase as the square root of the distance flown. It is hoped that when this analysis has been completed, it may be possible to ascertain these Final Errors from actual convoy positions, and correlate these errors with the navigation marks. In such an event it would be possible to predict what standard of navigation would have to be maintained in order to have odds of 9 to 1 of meeting a convoy at any required distance.

<u>Summary and Conclusion.</u>

(i) It has been shown that the standard method of analysis described in A.P.1456, and at present in use throughout Coastal Command, is

navigation maintained in the Command. It is recommended that it only be applied by the individual navigator to his own log on return from a sortie.

(ii) A simplified method of marking is described, and some applications of this system to convoy sorties, for the six month period ending 31st August, are given. It is shown that the standard of navigation within the Command can be readily assessed by this method, as well as its variations from squadron to squadron and with time. Other possible applications are indicated.

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13.10.42.

K.E.B.
H.H.P.

TABLE I :- Distribution at Navigation Marks.

Marks	No. logs	Marks	No. logs.
0.0 - 0.5	2	4.0 - 4.5	47
0.5 - 1.0	1	4.5 ⊖ 5.0	32
1.0 ⊖ 1.5	24	5.0 - 5.5	29
1.5 - 2.0	20	5.5 - 6.0	11
2.0 - 2.5	24	6.0 - 6.5	4
2.5 - 3.0	28	6.5 - 7.0	2
3.0 - 3.5	44	7.0 - 7.5	2
3.5 - 4.8			

Total No. Logs = 318
Mean mark......= 3.58
S.D............± 1.34

TABLE II:- Mean Marks per Month.

Month	Mean Mark	Month	Mean Mark
March	2.7	June	3.5
April	3.2	July	3.7
May	4.3	August	3.7

TABLE III:- Mean Marks per Squadron.

Squadron	No. Sorties	Mean Mark	S.D. at Mean
10	16	3.3.	± 0.26
120	59	4.0	± 0.13
160	7	5.2	± 0.39
201	92	3.6	± 0.14
220	39	3.3	± 0.17
228	54	3.4	± 0.21

TABLE IV:- Sorties to Convoys

			Navigation Mks.		Visibility	
Range	No.Sorties	%age Met	Met	Not-Met	Met	Not-Met
n.m.						
100-200	71	11.3%	3.5	3.8	7.0	7.3.
200-300	93	10.9	3.6	3.4	11.8	4.3
300-400	54	22.2	3.6	3.5	11.5	5.0
400-500	23	17.4	4.1	3.0	13.9	9.0
500-600	21	23.8	4.0	3.7	10.8	15.8
600	6	16.6	4.3	5.1	9.0	0.1

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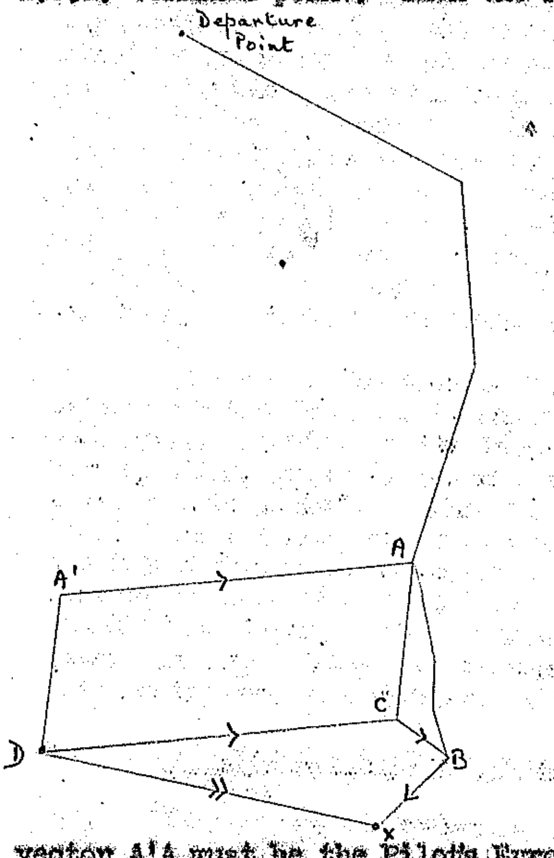

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120 Sqdn.	-	10 Sqdn. =	+0.7	± 0.29
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TABLE III:- Mean Marks per Squadron.

Squadron	No. Sorties	Mean Mark	S.D. of Mean
10	16	3.3	± 0.26
120	59	4.0	± 0.13
160	7	5.2	± 0.39
201	92	3.6	± 0.14
220	39	3.3	± 0.17
228	54	3.4	± 0.21

TABLE IV:- Sorties to Convoys

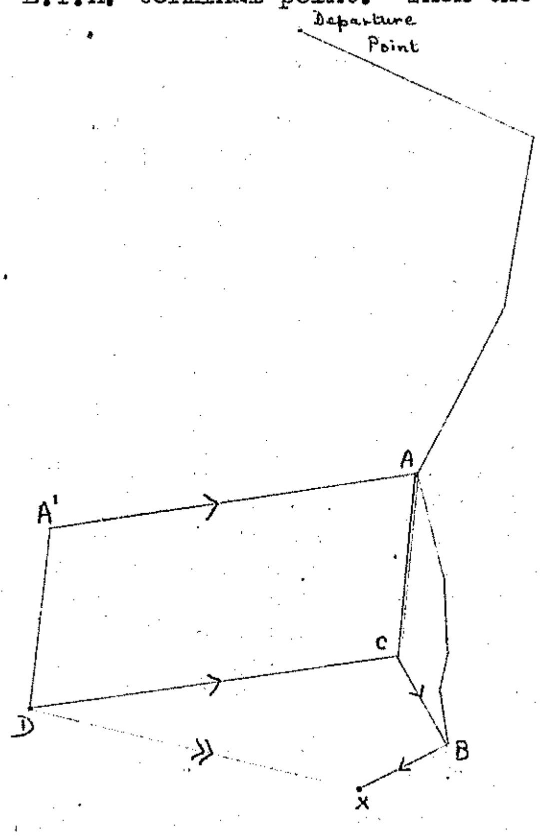
Range	No.Sorties	No%age Met	Navigation Mks.		Visibility	
n.m.			Met	Not-Met	Met	Not-Met
100-200	71	11.3%	3.5	3.8	7.0	7.3
200-300	93	10.9	3.6	3.4	11.8	4.3
300-400	54	22.2	3.6	3.5	11.5	5.0
400-500	23	17.4	4.1	3.0	13.9	9.0
500-600	21	23.8	4.0	3.7	10.8	15.8
600	6	16.6	4.3	5.1	9.0	0.1

ANALYSIS OF NAVIGATION LOGS.

To maintain a high standard of navigation within the Command, it is essential that the navigation in each long range sortie should be carefully assessed. Heretofore it has been the practice to carry out this assessment by means of the method of analysis laid down in A.P. 1456. While theoretically sound this method suffers from the grave disadvantages of being laborious and impracticable, particularly under operational conditions. Further it is applicable only to good navigation logs, and if mean results are taken from such analyses, these means tend to give an unrepresentative and too favourably biased a view of the standard of navigation within the Command. There is therefore clearly a need to consider other methods of assessing navigation logs, and it is the purpose of this report, after a brief consideration of the standard method of analysis, to describe one such method.

1. Standard Method of Analysis.

1.1. The method of analysis laid down in A.P. 1456 (Chap.XXX) is as follows:- Let X be the terminal point which the aircraft is required to reach, and D be the actual ground position of the aircraft at E.T.A. terminal point. Then the Final Error of the navigation is the vector distance DX, and the purpose of the analysis is to break this Final Error into its components. From the departure point the courses steered and the distances flown, as taken from the log, are accurately plotted up to point A, reached at E.T.A. terminal point. Note that A is not the air plot position corresponding to the ground position D reached at this time, for each of the legs which have been plotted leading up to A are subject to what may be called Pilot's Error, that is due to the failure of the pilot to maintain the courses and speeds specified by the navigator. To find this error lay off from D the true wind vectors reversed (shown in the figure as the straight line DA'), corresponding to the wind velocities which held throughout the area at the time the flight was made. The point A' thus reached must therefore be the air plot position corresponding to the ground position D, and the vector A'A must be the Pilot's Error.



Now lay off from A both the wind velocities used by the navigator (shown by the broken line AB), and the true wind velocities AC, where AC is equal and parallel to A D. Consequently DC (called in A.P. 1456 the Other Error) = A'A = Pilot's Error. The point B is the ground position which the navigator would have reached if his winds had been correct and if there had been no Pilot's Error. As, however, he wished to reach the terminal point X, the distance BX is the navigator's Calculation Error due to incorrect plotting, and calculation of courses to steer, while the vector CB is the error in the navigator's determination of wind (called in A.P. 1456 Wind Change Error). Consequently the Final Error DX = Vector Sum of DC (Pilot's Error), CB (Wing Error) and BX (Calculation Error), the sense of each vector being in the direction actual or true position to required position.

To effect this full analysis, which has broken the Final Error down

times over the route flown. The first of these can normally only be obtained from the landfall on the return to base, and this is only of use if the navigation has been continuously maintained throughout the whole sortie, including the critical period when the aircraft is homing on the A.S.V. beacon. Under operational conditions and over the sea a knowledge of the actual winds prevailing over the route is in effect impossible to obtain, particularly in view of the variability of the wind with time and place, which radio sonde ascents have revealed. Even that dubious approximation to the actual winds, namely the mean of the winds found by three or more aircraft flying over the same route at the same time, is rarely available under operation conditions.

1.2. The analysis of the Final Error into its components, as laid down in A.P. 1456, is therefore theoretically correct, and can be effected, provided the true winds and the true ground position are known. A moment's consideration, however, shows that the component errors resulting from the analysis are little more informative than the Final Error itself, Thus the Pilot's Error arises not only from the failure of the pilot to maintain the required courses and speed but also from instrument errors in the compass and A.S.I. which may not be accurately known, and / or not accurately applied by the navigator. In other words the Pilot's Error arises to a greater or less extent from failures of the navigator both on the ground and in the air. Similarly the Wind Error cannot be uniquely assigned to any particular part of the navigator's operations; thus winds determined by track and ground speed (or air plot) involve the Pilot's Error over the particular legs flown, as well as the errors of fixing the ground position, including errors of calculation, while multiple drift winds involve the accuracy of drift determinations, accuracy of piloting, and accuracy of calculation. Only the Calculation Error is unique, in that it does indicate what part of the Final Error is due to inaccurate plotting and calculation of courses to steer, but it only represents a small part of the computational errors which may be made by the navigator in the course of the flight.

In view of the uninformative nature of the analysis, the rarity with which the data can be obtained for accurately carrying it through and finally in view of its laboriousness, there seems little justification for squadron, group or command navigation officers continuing to analyse logs by this method. This is not to say that the analysis is without its use, but that use if for the individual navigator who has carried out the sortie. If he were required, following the sortie, to make the correct air plot and to find his so called Calculation Error, such an exercise, particularly if carried out under the supervision of his squadron navigation officer, could scarcely fail to improve the standard of this part of his work.

2. A System for Marking Logs.

2.1. The purpose of analyzing is to keep check on the standard of navigation within the Command or Group, and to discover means whereby that standard may be improved. A posible method of analysis would be to find the component errors introduced in to the Final Error as a result of each operation performed by the navigator. This has been unsuccessfully attempted in A.P. 1456, but this does not mean that it is impossible. To do it, however, would require special equipment and special personnel to be carried on sample operational flights throughout the Command, as well as a somewhat elaborate analysis of the results of such flights. Quite apart from the fact that such flights would ipso facto cease to be typical of the average operational flight, the amount of material gained in this way would perhaps be scarcely commensurate with the time and labour involved.

There is, however, another way of tackling this problem, When large numbers of operational flights are being made a relatively crude statistical

ample information as to the standard of navigation and the methods whereby it can be improved. Such a method consists in selecting from amongst the various operations performed by the navigator those for which evidence of performance can be inferred from the log. If the performance or non-performance of each operation is marked on some arbitrary scale, the sum of the marks so obtained is a measure of the standard of navigation, while if success or failure of the navigation is correlated with the marks given to each operation then some conclusions as to the value of that particular operation may readily be drawn.

2.2. One such system of marking is now being applied to all long range sorties to convoys for the six month period ending 31st August 1942. While the marking is not yet complete (there is a number of logs still to be received) and no analysis of the results has been made, it is perhaps worth while at this stage to indicate the method of marking and the kind of results which it may be expected to yield.

The perfect navigation log receives a score of 10 marks. These are assigned as follows:-

1 mark — for drift determinations made at the rate of 4 per hour throughout the navigational period.

3 marks — for wind velocity determinations made at the rate of 2 per hours. Wind velocities estimated from wind lanes and drifts are counted as half a determintion

1 mark — for accuracy of wind computation checked from data given in log on a Navigational Computer Mk.III The full mark is given if the wind given by the navigator agrees with the re-computation within 5 m.p.h. and $\pm$ 10°.

1 mark — for evidence and accuracy of changing from indicated to true air speed.

1 mark — for evidence of the use of the astro compass to check pilot's compass.

2 marks — for obtaining one astro position line per hour.

1 mark — for obtaining one loop bearing (or M.F./D.F. fix per hour).

These marks are entered on a card, one space for each mark and one card for a log. In addition the card carries the squadron number, the aircraft number and the names of the navigator and pilot; there are spaces for the day and times of flight, the distance from the last landfall to the predicted position of the convoy, a set of spaces for marking the search, and a summary of the meteorological conditions.

This method of marking has proved to be both objective and rapid. Thus a quarter of the logs have been marked independently by two computors, one with some practical experience of navigation and the other with none. In each case, apart from mistakes, the results have been in complete agreement. With regard to rapidity it is found that in so far as the required information can be obtained from the log itself, 15 minutes is sufficient to complete a card. Information which can only be obtained from Forms Orange, Group Narratives and from Convoy positions, and which is required for the analysis subsequently to be carried out, is not so easily or quickly obtained, and accounts for much of the delay experienced in completing the marking.

2.3. In all 318 logs have been marked, and the frequency distribution of the marks so obtained is shown in Table I (all tables appear at the end of the report). These marks only refer to the navigat ion carried

The mean mark for the 317 logs is 3.6 and the standard deviation of the distribution is ± 1.3. Since 7 of the possible 10 marks are given for a not unduly high standard of D.R. navigation, it will probably be agreed that some improvement in navigation throughout the Command is still attainable. There have in fact been changes throughout the period under review. Thus in Table II will be found the mean marks for each of the six months from March to August, and from this it can bee seen that the standard steadily increased from 2.7 in March to 4.3 in May, and then tapered off to a mean of about 3.6. Similar changes are shown in the navigation of eah of the six squadrons which did the bulk of this flying.

Not only does the mean mark vary with the time, but it also varies from squadron to squadron. The results are given in Table III for all squadrons taking part in seven or more sorties, excluding the Hudson squadrons. The first column of the table contains the squadron number, the second the number of sorties made, the third the mean mark for these sorties, and the final column the standard deviation of this mean. From this table we may readily take the differences between the mean mark obtained by 120 squadron and each of the other squadrons, as well computing the standard deviation of that difference. The results are as follows:-

120 Sqdn.	-	10 Sqdn.	=	+0.7 ± 0.29
120	-	160		-1.2 ± .41
120		201		+0.4 ± .19
120		220		+0.7 ± .21
120		228		+0.6 ± .24

The differences are in each case at least twice as large as the standard deviation, and may therefore be regarded as significant. There is therefore a real difference between the standard of navigation maintained in the various squadrons, and the standard of navigation within the whole Command would be greatly improved if each squadron could reproduce the results obtained by 160 Squadron. It is perhaps woth noting that it and 120 Squadron, the two highest scorers, are equipped with Liberators.

Finally it is of some interest to compare the navigation marks obtained for aircraft which successfully met and those which failed to meet convoys. In Table IV are given for various ranges the total number of sorties made, the percentage of "not-mets", and the navigation marks and visibility for the mets and not-mets. The results in this table confirm results earlier obtained in ORS/CC Report No.182, namely the increase in the percentage of not-mets with increasing distance from land, and the apparent dependance of meeting on visibility. However the numbers involved in Table V are so small, that means for the not-mets have little significance. Too much stress should not therefore be laid on the fact that the percentage of not-mets increases much more slowly with distance than found in the previous work, nor are the differences of visibility for meeting and not-meeting highly significant. With similar caution we note that there seems to be little correlation betwee the standard of navigation as given by the present system of marking, and failure to meet. The percentage increase of not-mets with distance is undoubtedly to be attributed to the Final Error of the navigation, and its probable increase as the square root of the distance flown. It is hoped that when this analysis has been completed, it may be possible to ascertain these Final Errors from actual convoy positions, and correlate these errors with the navigation marks. In such an event it would be possible to predict what standard of navigation would have to be maintained in order to have odds of 9 to 1 of meeting a convoy at any required distance.

Summary and Conclusion.

(i) It has been shown that the standard method of analysis described in A.P.1456, and at present in use throughout Coastal Command, is ineffective, laborious and non-representative of the standard of....

navigation maintained in the Command. It is recommended that it only be applied by the individual navigator to his own log on return from a sortie.

(ii) A simplified method of marking is described, and some applications of this system to convoy sorties for the six month period ending 31st August, are given. It is shown that the standard of navigation within the Command can be readily assessed by this method, as well as its variations from squadron to squadron and with time. Other possible applications are indicated.

ORS/CC.
13.10.42.

K.E.B.
H.H.P.

TABLE I :- Distribution at Navigation Marks.

Marks	No. logs	Marks	No. logs.
0.0 - 0.5	2	4.0 - 4.5	47
0.5 - 1.0	1	4.5 - 5.0	32
1.0 - 1.5	24	5.0 - 5.5	29
1.5 - 2.0	20	5.5 - 6.0	11
2.0 - 2.5	24	6.0 - 6.5	4
2.5 - 3.0	28	6.5 - 7.0	2
3.0 - 3.5	44	7.0 - 7.5	2
3.5 - 4.0	48		

Total No. Logs = 318
Mean mark......= 3.58
S.D............± 1.34

TABLE II:- Mean Marks per Month.

Month	Mean Mark	Month	Mean Mark
March	2.7	June	3.5
April	3.2	July	3.7
May	4.3	August	3.7

TABLE III:- Mean Marks per Squadron.

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500-600	21	23.8	4.0	3.7	10.8	15.8
>600	6	16.6	4.3	5.1	9.0	0.1